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THE ANTI-ROLL STABILIZATION OF SHIPS
BY MEANS OF ACTIVATED TANKS

PART C

SYNTHESIS OF HIGH PERFORMANCE SYSTEMS

BY

JOSEPH H. CHADWICK

ATI-174935

TECHNICAL REPORT NO. 15

PREPARED UNDER CONTRACT N6-ONR-251 TASK ORDER 2
(NR-041-943)
FOR
OFFICE OF AIR RESEARCH
AND
OFFICE OF NAVAL RESEARCH

DIVISION OF ENGINEERING MECHANICS

STANFORD UNIVERSITY

STANFORD, CALIFORNIA

MARCH 1951

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ABSTRACT

Part C presents a theoretical investigation of high-performance ship-stabilization systems, principally systems using U-tube tanks. Starting from the results of Part A, and using standard servo methods, we proceed to find systems with greater and greater effectiveness (for a given capacity). It appears that tank systems using inertia effects, may be as rapid in response as any other known stabilization systems. The results indicate that as long as the system's capacity is not exceeded, it should be possible to achieve greater than 90% stabilization. The three factors in ship stabilization: regulated elements; regulating elements; and the input, are discussed in turn, using appropriate servo methods and concepts.

This part of the report provides considerable grounds for believing that high-performance stabilization systems using tanks (and presumably other devices) can be achieved. It shows rational techniques by which such stabilization systems may be designed, and indirectly it emphasizes the inherent difficulties in design procedures which do not utilize servo techniques.

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I. INTRODUCTION

This is the third part of a four-part report, presenting the results of ship-stabilization research undertaken at Stanford University under the auspices of the Office of Naval Research and the Office of Air Research. Each of these parts treats a more or less distinct step in the research process, according to the following scheme:

THE ANTI-ROLL STABILIZATION OF SHIPS BY MEANS OF ACTIVATED TANKS

Part A -- Formulation of Problem and Basic Theory

Part B -- Verification of Basic Theory by Model System Tests

Part C -- Synthesis of High-Performance Systems

Part D -- Preliminary Considerations in the Design of Full-Scale Installations

The development of a linearized theory has been shown in Part A, and the evaluation of this theory shown in Part B. Part C is concerned with the design of high-performance ship-stabilization systems. The function of Part C, as the function of the whole research, is twofold. In the first place, it represents an attempt to find what the possibilities of stabilization using tanks may be, and how these possibilities may be achieved. It is also intended to demonstrate a general approach to the design of any ship-stabilization system: namely the general approach of modern servo theory.

In this problem, as in any other synthesis problem, there is no "absolutely perfect" solution. Nevertheless, the work that is recorded in this part of the report can be considered a search for, and an approach to, a system with optimum performance. As before, the methods and nomenclature used are rather similar to those of Brown and Campbell in <u>Principles of Servomechanisms</u> (New York: Wiley, 1948),[1] the principal emphasis being on the frequency-response approach and adjustment of the loop-transfer locus on the complex plane.

This is a mathematical, paper study, the results of which cannot be verified by the very non-optimum experimental apparatus available at Stanford. Attention has been confined to fundamental questions, which provide plenty of subject matter in any event, and do not require an untoward extrapolation of the basic theory. As this is a mathematical study, only a moderate amount of attention is given to the limits (i.e. the capacities) of the lesser elements of the system. Only those physical limitations which are of first magnitude importance are discussed here. The question of limits is taken up in detail in the fourth part of this report, Part D.

Before the problem can be attacked profitably it must be condensed to its simplest and most basic form. This is done

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[1] Oppelt, <u>Stetige Regelvorgänge</u>, and James, <u>Theory of Servomechanisms</u>, are also followed to some extent.

in Chapter II. As in the preceding parts of the report, we distinguish three factors: (1) regulated elements; (2) regulating elements; and (3) the input. These factors are considered in Chapters III, IV, V, respectively. In Chapters III and IV, a series of fundamental questions in ship-tank design and in control design are studied quantitatively, using the frequency-response approach. In Chapter V, the use of waveslope information is discussed in a brief and general way and related to the statistical RMS-error approach to servo design.

II. STATEMENT AND CONDENSATION OF THE PROBLEM

A. Basic Theory From Part A

We might best begin by bringing forward the theory developed in Part A; i.e. the servo analogy, the servo equations, the differential equations of motion, and the system block transfer-responses, $\underline{B}$ and $\underline{C}$. Recall from Part A that the stabilized ship is analogous to the following <u>motion</u> servo-mechanism:
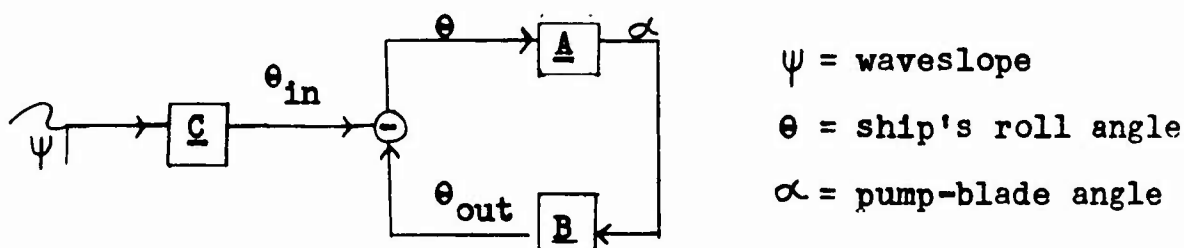
Figure 1. Stabilized Ship as a Servomechanism

where:

ψ = waveslope

θ = ship's roll angle

α = pump-blade angle

A number of servo equations may be derived from Figure 1. First, when the controls are operating, one obtains the actively stabilized response, which in our case might be called the <u>Minorsky-stabilized response</u>:

$$\frac{\theta}{\psi} = \frac{\theta_{in}}{\psi} \times \frac{\theta}{\theta_{in}} = \underline{C} \left(\frac{1}{1+\underline{A}\ \underline{B}}\right) = (\text{Frahm stab.}) \times (\text{Feedback stab.})$$

Second, if the control loop should now be opened so that $\underline{A}$ = zero, one has what in our case may very properly be called the <u>Frahm-stabilized response</u>:

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$$\frac{\Theta}{\psi} = \frac{\Theta_{in}}{\psi} \times \frac{\Theta}{\Theta_{in}} = \underline{C}\left(\frac{1}{1}\right) = \text{(Frahm stab.)}$$

A third response may be defined as a standard reference, namely the response with the control loop open and the tank duct blocked. This is obviously the unstabilized response:

$$\frac{\Theta}{\psi} = \frac{\Theta_{in}}{\psi} = \underline{C}\ \bigg|_{\text{blocked duct}} = \text{unstabilized response}$$

In physical terms, one obtains the standard unstabilized response (as used hereafter) when the tank ducts of the ship are solidly blocked; the Frahm-stabilized response when the control loop is open, the pump motors are turning, but the tank ducts are not blocked; and the Minorsky-stabilized response when the pump motors are turning and the control loop is closed. Interestingly, the Minorsky (or overall) stabilized response is seen to be the product of the Frahm-stabilized response and the standard feedback factor for an elementary servo loop.

As we are interested only in general results, we need only the non-dimensional equations of motion. From Part A, these are (in operator notation):

Ship: $[P^2 + \frac{P}{Q_s} + 1]\ \theta + \lambda_t[(\frac{P}{\Omega_{st}})^2 + 1]\ \varphi = \lambda_{ss}\ \psi$

Tank: $\lambda_t[(\frac{P}{\Omega_{st}})^2 + 1]\ \theta + \lambda_t[(\frac{P}{\Omega_t})^2 + (\frac{P}{\Omega_t})\ \frac{1}{Q_t} + 1] = \lambda_p\ \alpha$

- 5 -

Where,

$$\Omega_t = \frac{\omega_t}{\omega_s} = \text{relative resonant frequency of tanks}^2$$

$$\Omega_{st} = \frac{\omega_{st}}{\omega_s} = \text{relative frequency of secondary resonance}$$

$$Q_s = \frac{\omega_s J_s}{B_s} = \text{normalized ship damping factor}$$

$$Q_t = \frac{\omega_t J_t}{B_t} = \text{normalized tank damping factor}$$

$$\lambda_t = \frac{K_t}{K_s} = \text{relative tank strength parameter}$$

$$\lambda_{ss} = \frac{K_{ss}}{K_s} = \text{relative wave strength parameter}$$

$$\lambda_p = \frac{K_p}{K_s} = \text{relative pump strength parameter}$$

$$P \equiv \frac{d}{dt'} = \frac{1}{\omega_s} \frac{d}{dt} \; ; \quad \varphi = \text{tank water angle}$$

Remembering that, $\underline{B} = \frac{\theta_{out}}{\alpha}$ and $\underline{C} = \frac{\theta_{in}}{\psi}$, it quickly follows from the above that,

$$\underline{B} = \frac{\lambda_p [ST]}{[SHIP][TANK] - \lambda_t [ST]^2} \; ; \text{ and } \underline{C} = \frac{\lambda_{ss}[TANK]}{[SHIP][TANK] - \lambda_t [ST]^2}$$

SHIP-TANK TRANSFER RESPONSES

where by definition,

---

[2] For a more complete definition of these equations and parameters, see Part A of this report.

$$[SHIP] \equiv [P^2 + \frac{P}{Q_s} + 1]$$

$$[TANK] \equiv [(\frac{P}{\Omega_t})^2 + (\frac{P}{\Omega_t}) \frac{1}{Q_t} + 1]$$

$$[ST] \equiv [(\frac{P}{\Omega_{st}})^2 + 1]$$

The exact definition of the control block transfer-response, $\underline{A}$, is left open for the moment; it will have the general form of a gain constant, K, times some function of P.

B. Four Critical Ship-Tank Parameters

In the paragraphs above, $\underline{B}$ and $\underline{C}$ are written in terms of seven non-dimensional parameters. This is less than the original nine dimensional parameters, but still would seem to represent an almost prohibitively complex parameter space. Fortunately, only four of these seven parameters are essential to the mathematical treatment of stabilization. Because the damping of the ship is small, it has a negligible effect on the stabilized performance and may be disregarded (assume Q_s = infinity). Because it is exterior to the whole system and simply defines the effective strength of waves, λ_{ss} may be specified equal to unity. Because it acts in series with the control system, $\underline{A}$, and may be combined with the control gain constant, K, λ_p may also be specified equal to unity. Note that these last two steps do not represent approximations, but actually have no essential effect on the linearized mathematics of performance.

This reduces our parameter space to a space of four dimensions. Mathematically, these parameters are "free variables", but actually there are considerations of practicality entirely apart from the mathematics which limit the ranges of some of them. There are also economic reasons for making certain parameters as large or as small as possible, etc. These critical parameters are discussed one by one, below.

Relative tank strength parameter, λ_t: For a given motion of the fluid in the tanks, the torque on the ship is directly proportional to λ_t. For a given shape of the tanks and ducts, the per cent weight of water on the ship is also directly proportional to λ_t. Because the static stability of the ship is $(1 - \lambda_t)$, λ_t cannot be allowed to exceed unity, and for safety's sake should not exceed say about 0.50. One of the objects of this study is to find the smallest λ_t, and hence the least weight of fluid, compatible with the desired stabilization.

Relative resonant frequency of tanks, Ω_t: As the ship is a highly resonant system, it is clear that Ω_t must be reasonably close to unity if the tanks are to operate in the conventional manner. It is known that for best Frahm stabilization, Ω_t should be equal to unity. For operation in the usual manner, it probably should not range outside the limits, 1 ± 0.30.

Relative frequency of secondary resonance, Ω_{st}: This is the relative frequency where the torque exerted by the tanks becomes zero and reverses. For this reason Ω_{st} must be _above_ unity, say at least 1.5 or greater. It is possible for Ω_{st} to be infinite or imaginary, but this involves raising the tanks in the ship and has definite practical limitations. The higher the tanks are placed in the ship, the higher the frequency of secondary resonance, but it will probably not be possible to go much beyond $\Omega_{st} = 3.0$.

Normalized tank damping factor, Q_t: This parameter has the greatest free range of all the parameters. It appears possible to design systems with Q_t ranging from about 3.0 to 0.1, which includes the whole range of mathematical interest. We also are not aware of any _a priori_ mathematical arguments sufficiently powerful to narrow this range. It therefore seems desirable to begin our study by narrowing the range of practical or desirable Q_t, which may be done by studying performance as a function of this parameter.

C. Various Possibilities in Ship-Tank and Control Design

The following two chapters consist principally of a series of studies of interesting special systems, each of which illustrates some fundamental question in the problem of stabilization by tanks. Chapter III deals principally with ship-tank possibilities, and Chapter IV principally with control possibilities.

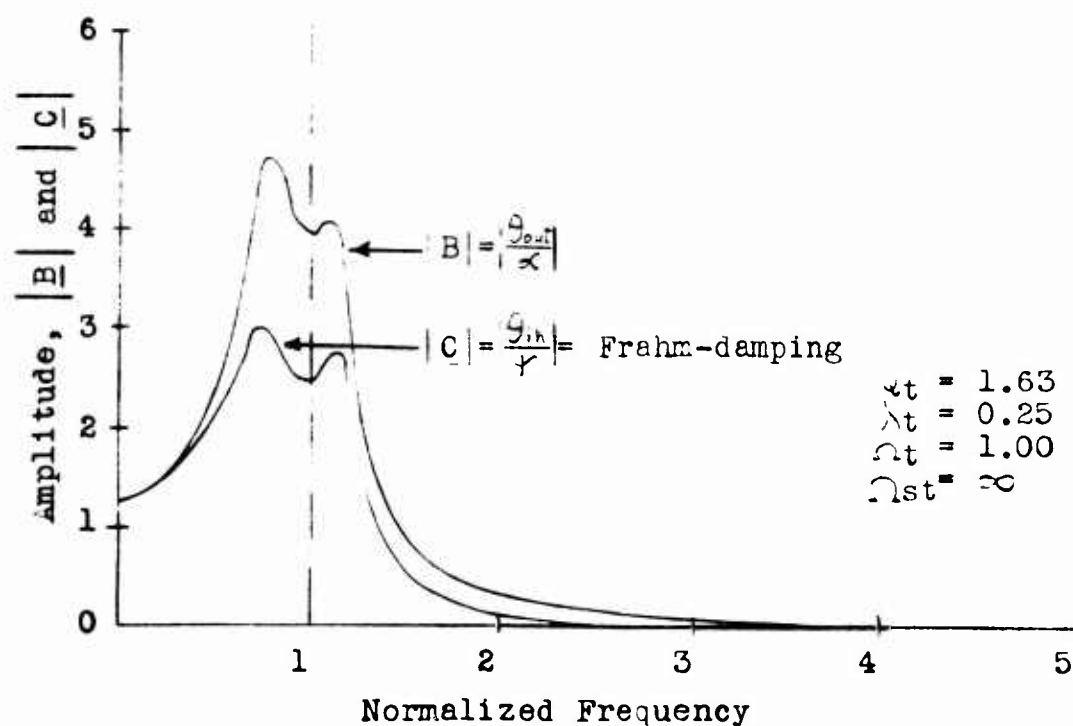Figure 3. Open-loop transfer functions $\underline{B}$ and $\underline{C}$ for the ideal Frahm-damping case.
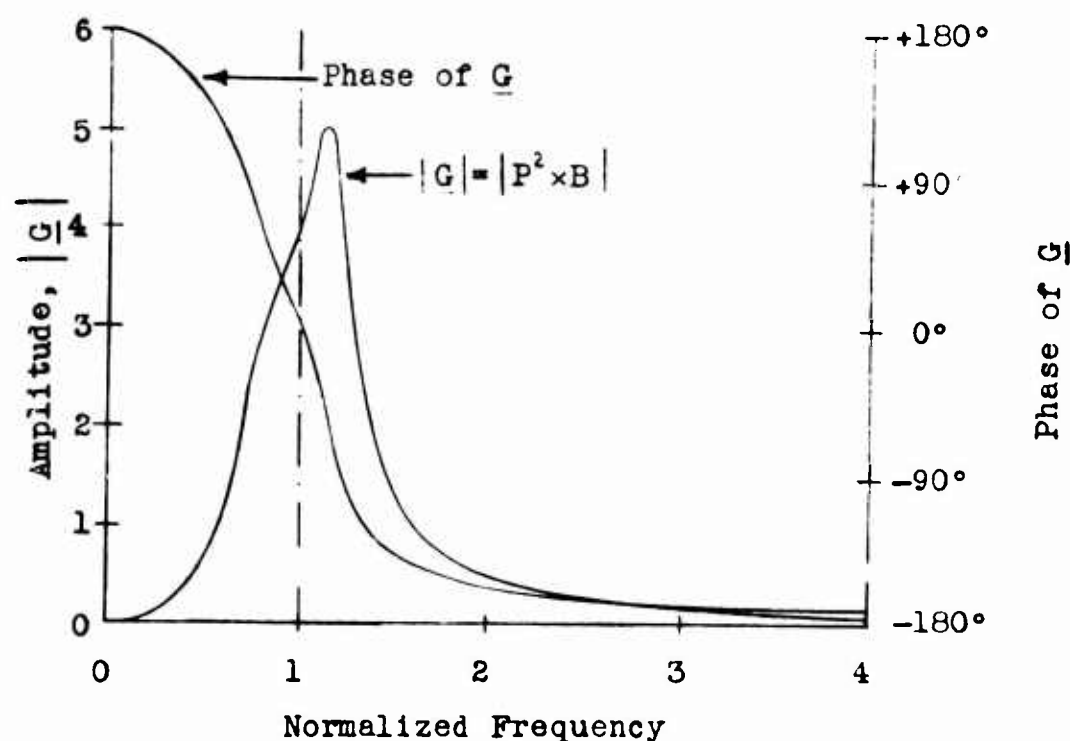
Figure 4. Frequency variable part of the overall transfer function, $\underline{G}$ function.

Q_t = 1.63

λ_t = 0.25

Ω_t = 1.00
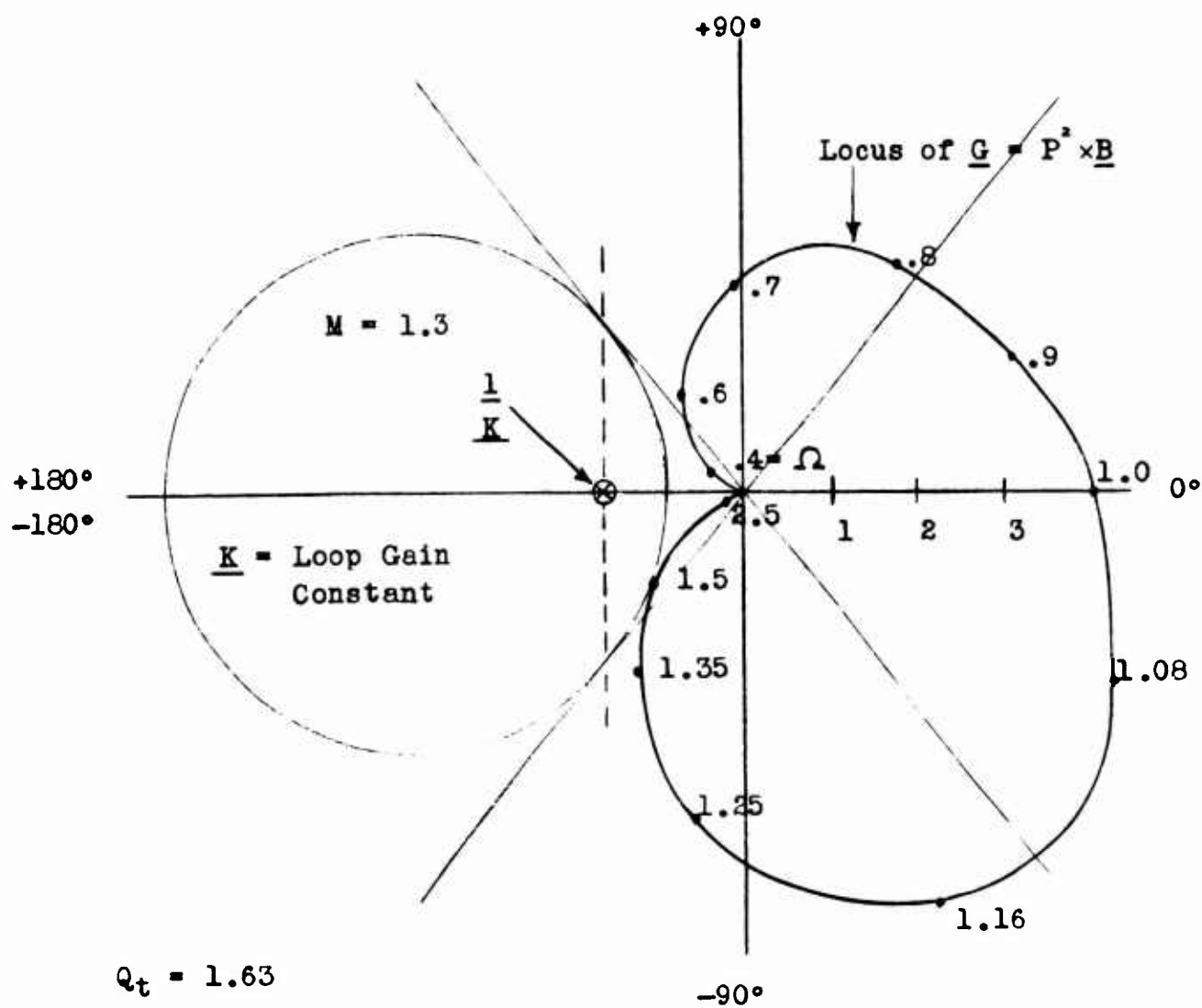
Ω_{st} = ∞

$\underline{K}$ = 0.64

Figure 5. Determination of $\underline{K}$ using the Nyquist Diagram, for the
ideal Frahm-damping case.

Ship-Tank Possibilities: We are considering ships with one or more sets of U-tube tanks on board. If all the tanks on a given ship have the same parameters, and if all their pump-blade angles move together, the system may be treated as a single tank system. For a single tank system the totality of ship-tank possibilities is the set of all systems with all practical combinations of the four parameters. That is, the set of systems occupying all those points of the four-parameter space which may be realized in practice. It would be undesirable as well as impossible to study every point of this space. It is perhaps more profitable to approach the problem with a topological viewpoint, to try to determine what regions, points, parameters, etc., are most critical to performance, and then to concentrate on these.

It is possible to imagine systems in which there are a number of U-tube tanks with different parameters, and such systems appear to offer opportunity for further performance improvement. Certain very fundamental questions in the theory of stabilization by tanks may be illustrated using such diversified tank systems, as the reader will find in succeeding sections.

Control Possibilities: The Minorsky system of "activated tank" stabilization uses an accelerometer to provide the basic signal which controls the pump-blade angle. While acceleration should indeed be the principal signal controlling the

blade angle, it alone cannot provide a stabilizing action at zero frequency. One is naturally led to study the use of subsidiary signals proportional to velocity and position. These signals, which in a sense are integrals of acceleration, would be used to improve the low-frequency response. The high-frequency response may possibly be improved by certain tank arrangements involving the rational use of secondary resonance, and may be further improved by the use of "lead circuits", et al. Finally there is a very interesting possibility in the use of "feedahead",[3] in addition to the conventional "feedback".

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[3] See for example, R. E. Graham, "Linear Servo Theory", BSTJ, October 1946, who calls this type of control, feed-forward. "Feedahead" was also mentioned in Part A (Chapter V, Section A).

III. FREQUENCY-RESPONSE APPROACH TO SHIP-TANK DESIGN

This chapter is concerned with the design of high-per-
formance regulated elements. We shall systematically study
the effects of varying the four critical parameters of the
single tank system, and also study the possibilities of various
diversified tank systems. To divide the difficulties, the con-
trol transfer-response will be held standard while we deliber-
ately vary the ship-tank transfer-responses. For this chapter
assume the control system to be an ideal accelerometer in
series with an ideal amplifier, so that the pump-blade angle is
at all times proportional to the roll acceleration of the ship.
From this one derives the following definition for $\underline{A}$,
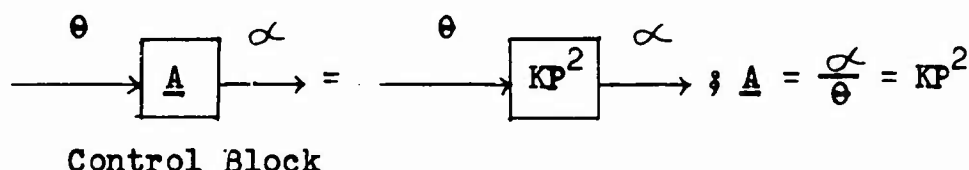
Control Block

Figure 2. Control Block Transfer-Response

A. Outline of Method

General: The method used here is essentially identical
to that given in Brown and Campbell, Chapters 7 and 8. Having
assumed a set of ship-tank parameters, we can calculate $\underline{B}$ and
$\underline{C}$ as functions of P. We can also calculate the total transfer-
response around the closed loop, or loop-transfer character-
istic, K $\underline{G}$. By definition,

$$K \underline{G} \equiv \underline{A} \times \underline{B} \quad \text{and} \quad \underline{G} \equiv \frac{\underline{A} \times \underline{B}}{K}$$

For the control response assumed in this chapter,

$$\underline{G} \equiv \frac{A \times B}{K} = P^2 \times \underline{B}$$

Now all that remains is to find the best value for the <u>control gain factor</u>, K, for each case. This is done graphically with the aid of a Nyquist plot of $\underline{G}$ (i.e. the locus of $\underline{G}$ as a function of frequency on the complex plane). To determine this locus P may be defined as complex frequency; where for real frequency, $P = j\Omega = j\frac{\omega}{\omega_s}$.

Generally, for effective stabilization, K must be as large as possible short of the point where transients begin to persist in the system due to excessive gain. As this point is rather critical, K is fixed to all intents and purposes by the original choice of parameters and need not be considered a free variable in the problem. In all the examples of this chapter, K will be chosen in accordance with the criterion, M = magnification ratio = 1.3. The <u>magnification ratio</u> is a measure of the peaking up of the response of the stabilized system in the vicinity of the most critical frequency, and hence a measure of the damping of the system's most critical normal mode.[4]

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[4] For a more detailed explanation of this process see Brown and Campbell, <u>Principles of Servomechanisms</u>, Chapter 6.

List of Steps: The standard procedure used in all cases is as follows:

1. Select the desired ship-tank parameters.

2. Calculate the two transfer functions, $\underline{C} = \dfrac{\theta_{in}}{\psi}$, and $B = \dfrac{\theta_{out}}{\alpha}$. Note that the phase of $\underline{C}$ is immaterial, as $\underline{C}$ is external to the closed loop and the phases of incoming waves of different frequencies are essentially incoherent.

3. Calculate the loop-transfer characteristic, $\underline{G} = P^2 x \underline{B}$.

4. Plot $\underline{G}$ on a Nyquist diagram, and then find the best value for K by the criterion $M = 1.3$.

5. Calculate $\dfrac{\theta}{\theta_{in}}$ = feedback stabilization = $\dfrac{1}{1+\underline{A}\ \underline{B}} = \dfrac{1}{1+KP^2\underline{B}}$.

6. Calculate $\dfrac{\theta}{\psi}$ = overall stabilization = $\dfrac{\theta_{in}}{\psi}\dfrac{\theta}{\theta_{in}} = \underline{C}\ (\dfrac{1}{1+\underline{A}\ \underline{B}})$.

7. Compare the unstabilized, Frahm-stabilized, and Minorsky-response of the ship to waves of various frequencies.

8. Compare these responses to those obtained for other values of the ship-tank parameters.

Presentation of Figures: In order to keep the number of figures from getting out of hand, we shall show this whole process through once, and afterward include only those curves which directly illustrate a point of interest. Most of the curves presented are either plots of stabilized response versus frequency or Nyquist diagrams of the loop-transfer locus, $\underline{G}$. On all figures showing stabilization versus frequency, the unstabilized response is plotted as a reference.

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B. Best Parameter Values for Single Tank System

In the preliminary work we found that the <u>mathematically</u> important characteristics of a <u>single tank system</u> could be expressed in terms of four parameters: Q_t, λ_t, Ω_t, Ω_{st}. Let us now attempt to determine the effect of each of these parameters on system performance, and by this means gain a rational and complete understanding of the single tank system.

<u>Best Value of Q_t</u>: As we know least about the effect of Q_t, it is studied first. With the other parameters at reasonable values, a series of values for Q_t may be investigated. Because the <u>Frahm-stabilized response</u> is a factor in the overall (<u>Minorsky-stabilized</u>) response, it is plausible to suppose that optimum Minorsky-stabilized response might be associated with optimum Frahm response. This leads us to begin with the value of Q_t which gives the best Frahm stabilization in the sense of Den Hartog, $Q_t = 1.63$.[5] We arbitrarily set $\Omega_t = 1.0$, $\Omega_{st} =$ infinity and $\lambda_t = 0.25$. The results of the step-by-step process described in the preceding section are shown in Figures 3, 4, 5, 6, and 7.

Surprisingly, the <u>overall</u> stabilization in this case is seen to be very poor. As the experimental system shows better

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[5] Assuming the value of $\lambda_t = 0.25$, the "best" values for Q_t and Ω_t may be found by a method exactly analogous to that used by Den Hartog, <u>Mechanical Vibrations</u> (2nd Ed.), (New York: McGraw-Hill, 1940) pp. 115-29.
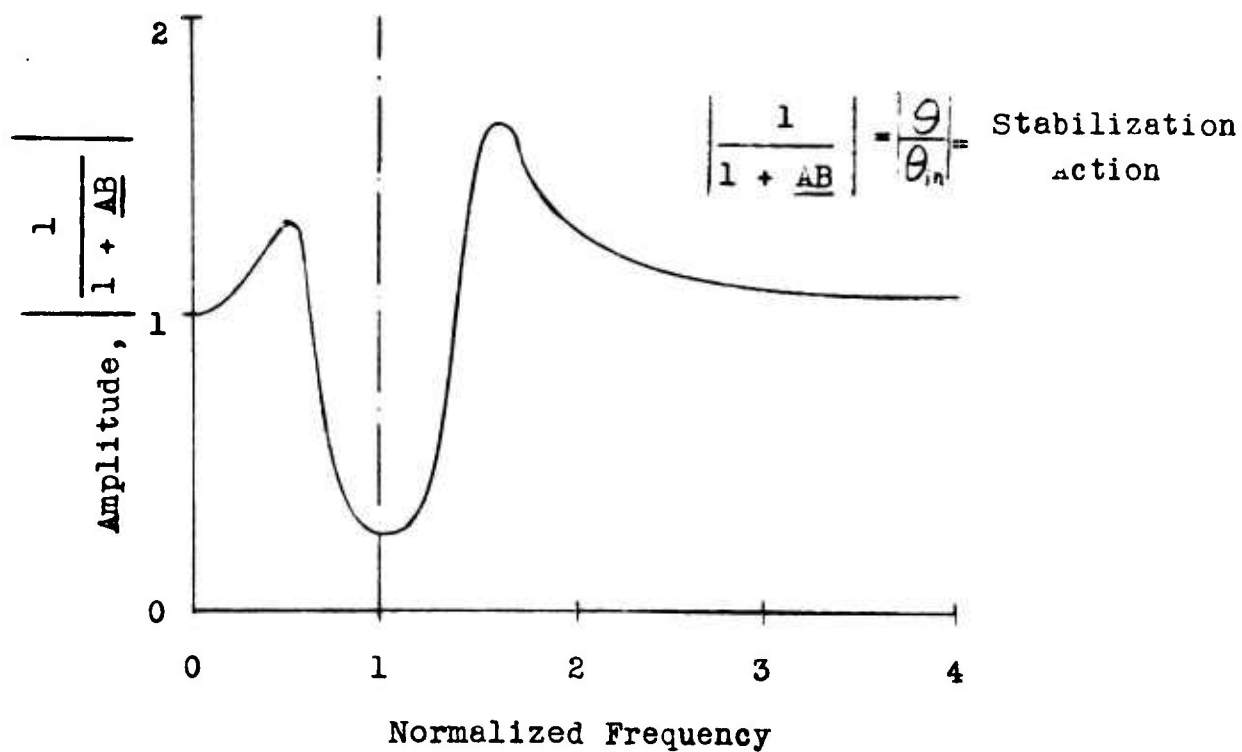
Figure 6. Stabilization action for the ideal Frahm-damping case.
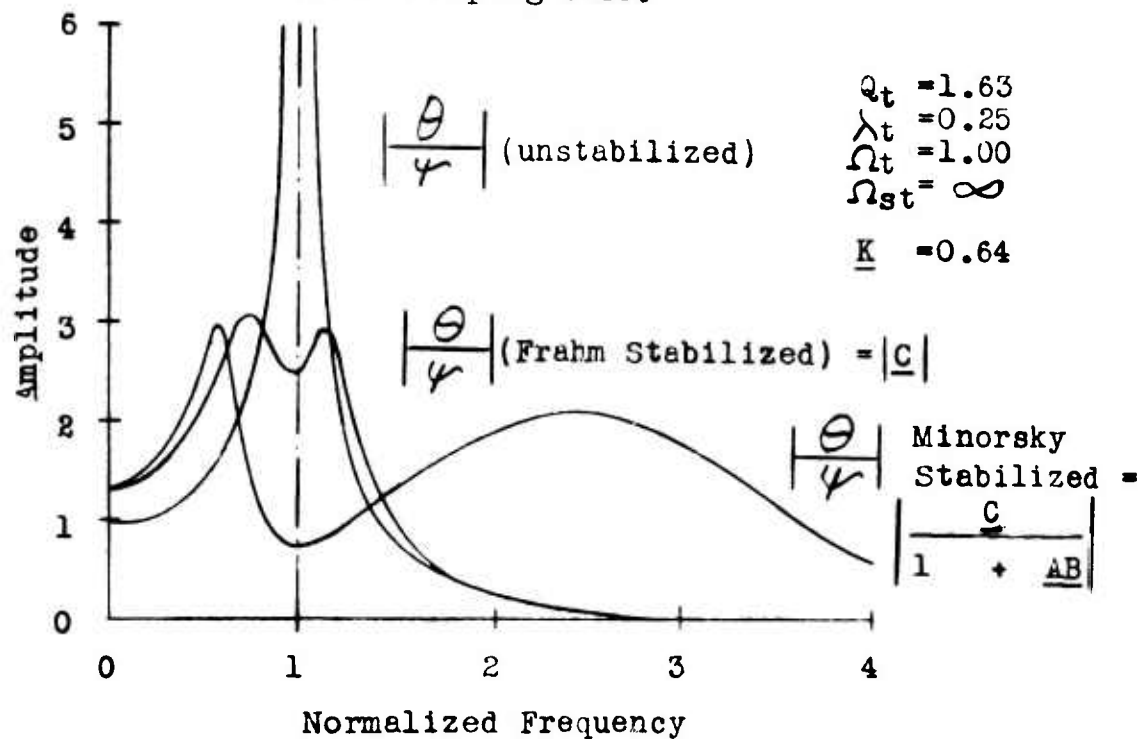
Figure 7. Unstabilized, Frahm-stabilized, and Minorsky stabilized response for the ideal Frahm-damping case.

stabilization than this and has lower Q_t, we next choose a series of decreasing values for Q_t, with the results shown in Figures 8 and 9. It is clear that, under the specified conditions, the excellence of stabilization increases monotonically with decreasing Q_t. There is a practical limit to this process, however, in that the actual realization of Q_t's less than 0.1 would probably be quite difficult, and would also lead to conditions more complex than those described by the theory we are presently using. The process has been stopped at $Q_t = 0.2$. but the tendency is clear.

Least Possible Value of λ_t: As λ_t is proportional to the per cent weight of water on the ship, it is advantageous from an economic point of view to make it as small as possible. In practice, the reduction of λ_t will be limited by the fact that reducing it tends to reduce the maximum capacity of the system. We wish to investigate here, however, whether or not the reduction of λ_t will have an adverse effect on performance. assuming that the capacity of the system (however large or small. is never exceeded. The results of such a reduction are shown in Figures 10 and 11. It appears that the value of λ_t (in itself) has very little effect upon the excellence of stabilization, and if anything the stabilization improves slightly with decreasing λ_t. Because a smaller value of this parameter means that the water must travel further to counteract a given waveslope, it will probably not be possible to design U-tube type tank

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G - Plane

Low Freq. ⟶

Center Freq.

+180°
−180°

0°

1   2   3

High Freq. ⟶

$\lambda_t = 0.25$

$\Omega_t = 1.00$

$\Omega_{st} = \infty$

(1) $\lambda_t = 1.63$

(2) $\lambda_t = 1.00$

(3) $Q_t = 0.50$
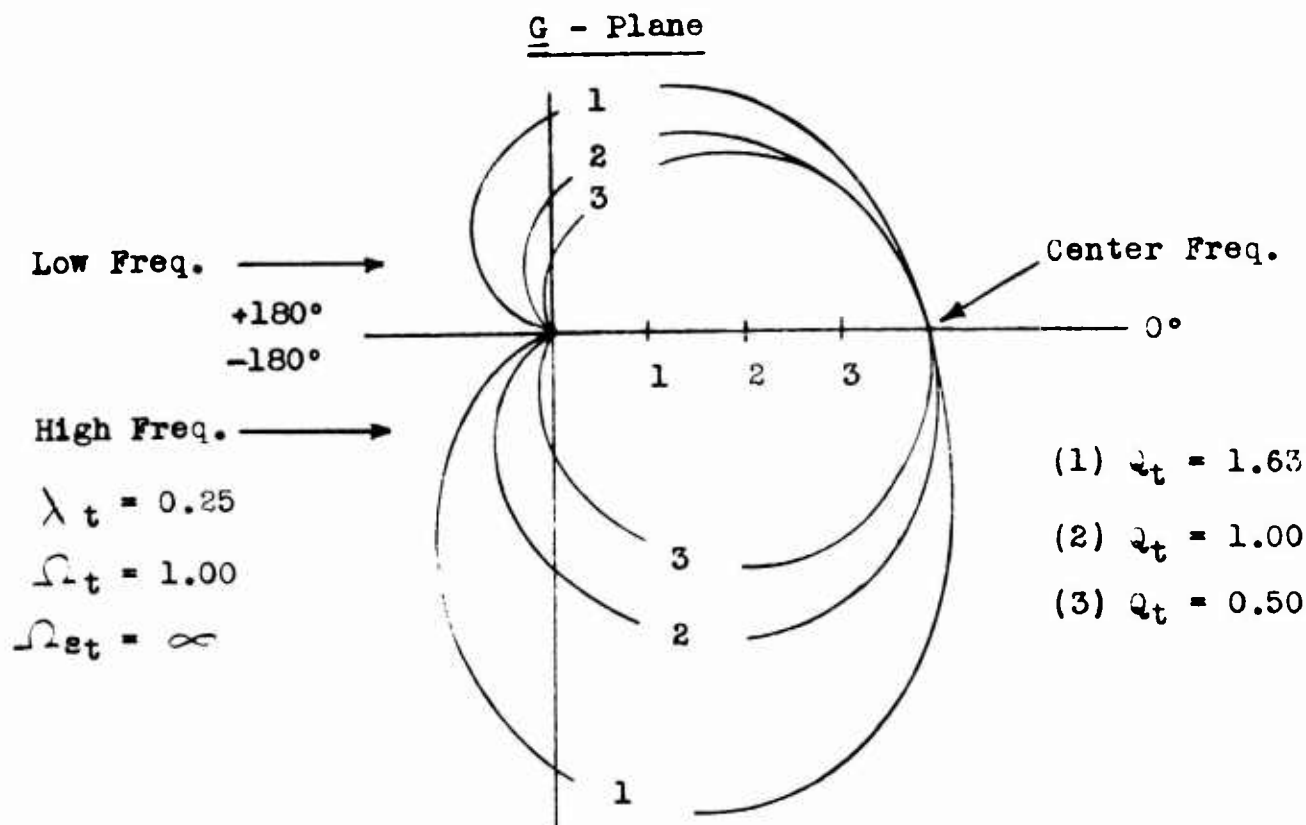
Figure 8. Nyquist Diagrams - Change in the G locus with decreasing Q_t.

$\left|\dfrac{\Theta}{\psi}\right|$ (unstabilized)

(1) $Q_t = 1.63$; $K = 0.64$

(2) $Q_t = 1.00$; $K = 1.52$

(3) $Q_t = 0.50$; $K = 4.00$

(4) $\lambda_t = 0.20$; $K = 20.0$

$\left|\dfrac{\Theta}{\psi}\right|$ (stabilized) $= \left|\dfrac{G}{1 + AB}\right|$

Amplitude

Normalized Frequency
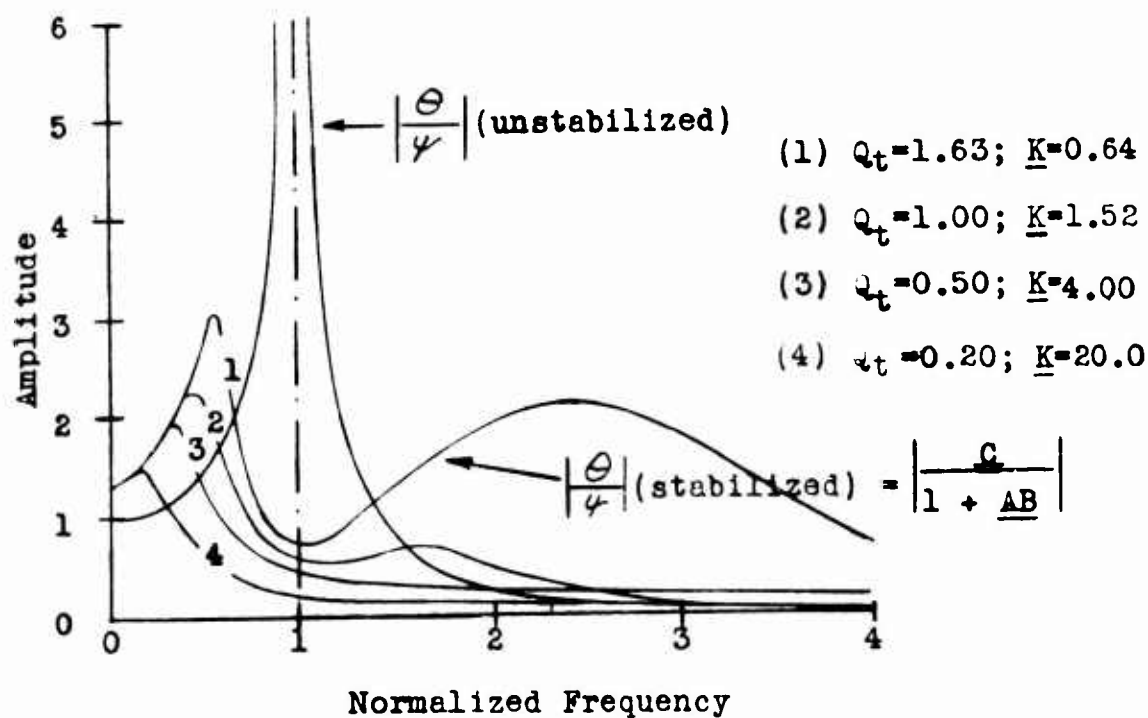
Figure 9. Improvement of stabilization with decreasing Q_t.

systems with λ_t less than 0.10.[6]

Best Value of Ω_t: It can be shown that for optimum Frahm-stabilization, $\Omega_t = 1.0$. Figures 12 and 13 show the effects of slightly larger and slightly smaller values of Ω_t on the loop-transfer locus and on Minorsky-stabilized performance. It is apparent that as Ω_t is decreased, the low-frequency response becomes better, the high-frequency response becomes poorer, and vice versa as it is increased. As it is easier to improve the low-frequency stabilization by special controls than it is to improve the high-frequency stabilization, a value of Ω_t in the neighborhood of 1.30 is probably pretty close to optimum. However, if it is intended to use the tanks in some cases as simple Frahm dampers, Ω_t should be left at 1.0.

Best Value of Ω_{st}: This is in some ways, one of the most important and interesting of the tank parameters, as has been mentioned before. At the point where $\Omega = \Omega_{st}$, a single tank system will produce no net stabilizing torque, no matter how great the motion of water in the tanks. It is therefore desirable to have Ω_{st} as high as possible in a single tank system. Again there is a practical limitation. It will undoubtedly be difficult to achieve Ω_{st}'s much higher than 3.0 in practice. This represents a serious limitation on the utility of the single tank system (of the U-tube type). Certain ways to

---

6 For the USS Hamilton installation designed by Minorsky, our calculations show that $\lambda_t \cong 0.25$.

G – plane

+90°

1
2
3

$Q_t = 0.20$

$\lambda_t = 0.25$

$\Omega_{st} = \infty$

(1) $\Omega_t = 0.80$

(2) $\Omega_t = 1.00$

(3) $\Omega_t = 1.20$

+180°
−180°

0°

0.1   0.2

1
2
3

−90°
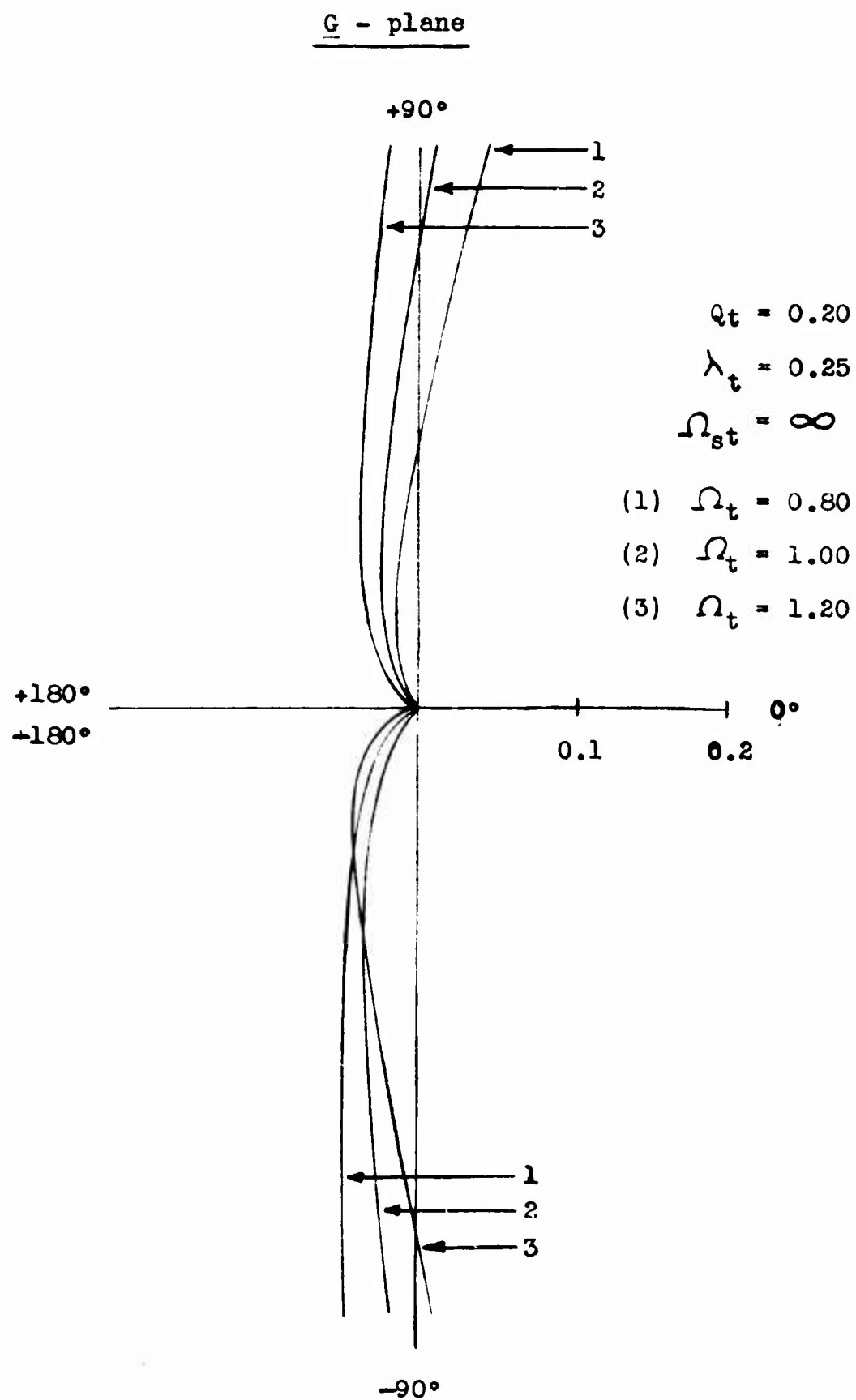
Figure 12. Nyquist diagrams – Change of G locus with increasing
and decreasing Ω_t.
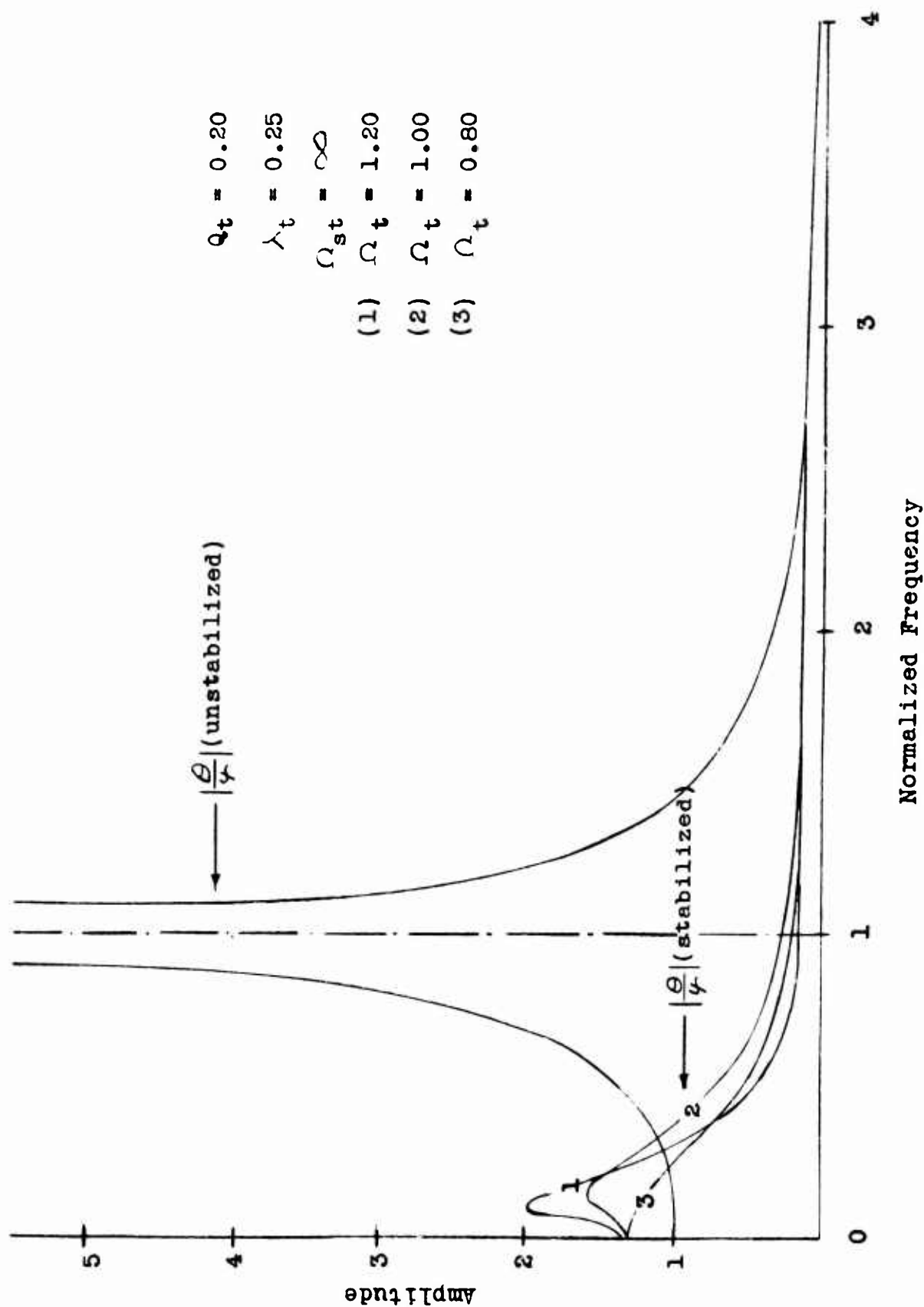
Figure 13. Comparative stabilization for various Ω_t.

circumvent this difficulty will be discussed in the following sections. The effect of $\Omega_{st} = 2.0$ is shown in Figures 14, 15, and 16. Note that at $\Omega = \Omega_{st}$ the phase of the stabilization action goes through zero and _reverses_.

C. Diversified Tank Systems

Diversification of the regulated elements is a feature of many high-performance servo systems. It is not, of course, an end in itself. One has the right to expect rather marked improvement in performance as justification for the added complexity. In general, one must begin with a concrete idea of what is to be accomplished by the diversification. Two _diversified tank systems_, corresponding to two such specific ideas, appeared beforehand to offer promise and are discussed below. They are: (1) two equal-sized but "stagger-tuned", tanks; and (2) a single tank, with a small "helping" tank to fill in the secondary resonance null.

Stagger-Tuned Tanks: The modus operandi of "stagger-tuned" tanks is quite simple. Instead of a single U-tube tank, tuned approximately to the ship's natural frequency, we could use two U-tube tanks of equal size, one tuned slightly above and one tuned slightly below the natural frequency of the ship. The hope here is that the "broad-banding" produced by this arrangement will materially improve the loop-transfer characteristic, and hence the stabilized performance. Actually, this possibility was conceived before it was known that the

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G - plane

Note: Action is now, in the stabilizing sense, at high frequency.

$\Omega = \Omega_{st}$
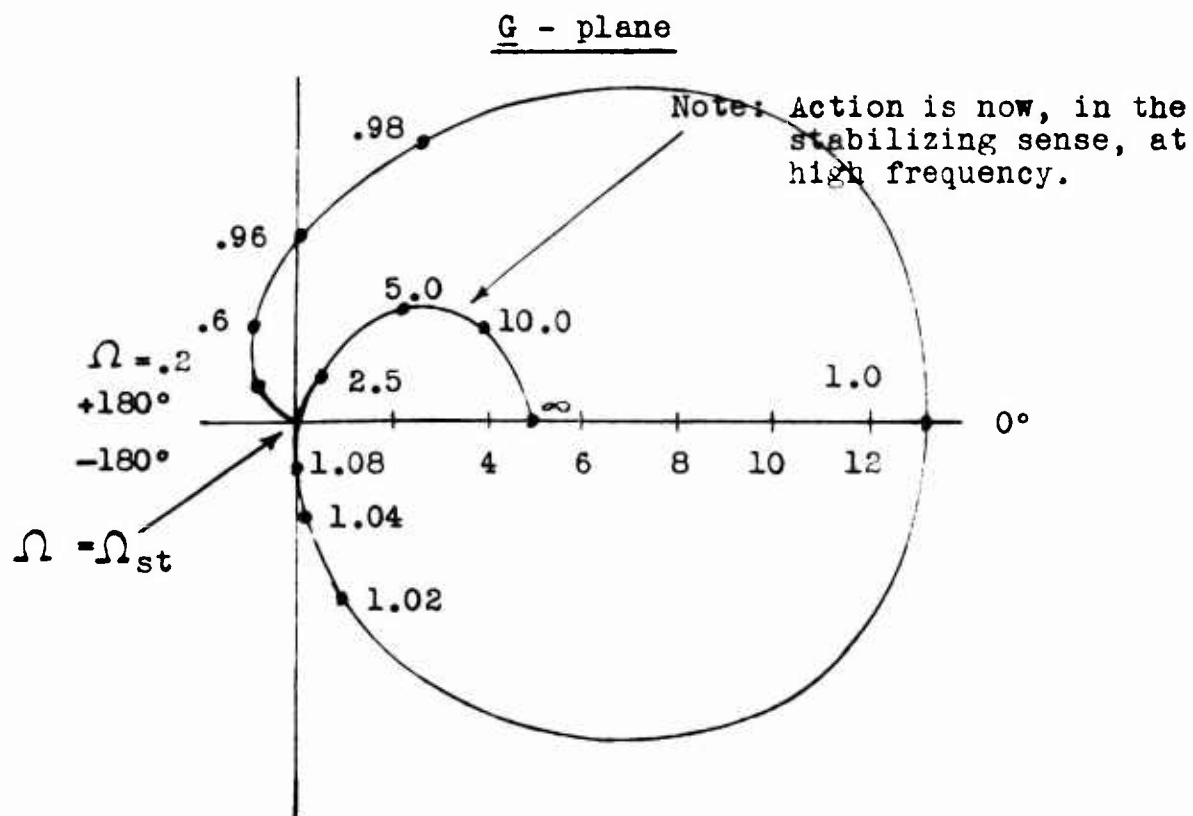
Figure 14. Nyquist Diagram - Exaggerated effect of secondary resonance on the Nyquist diagram.

$Q_t = 0.20$

$\lambda_t = 0.10$

$\Omega_t = 1.00$

$\Omega_{st} = 2.00$
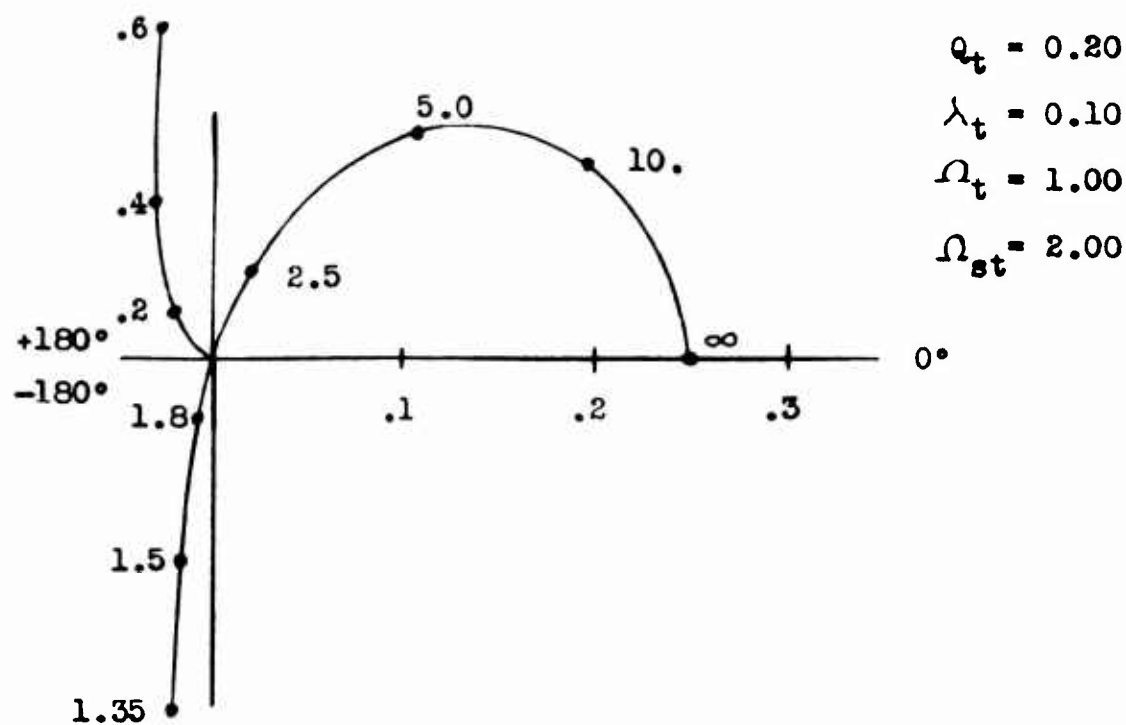
Figure 15. Actual effect of secondary resonance, $\Omega_{st} = 2.0$.

Figure 16. Effect of finite Ω_{st} on stabilization.

best <u>single tank system</u> would have a very low value of Q_t. With such a low Q_t in the single tank system, the further "broad-band-ing" produced by "stagger-tuning" does not have a significant effect, as can be seen from the stabilization curves in Figure 17.

<u>Single Tank with Small Helping Tank</u>: In some servomech-anisms, the principal regulated element has a weak spot in its frequency response which may be eliminated or reduced by the addition of a small subsidiary device which "helps" in this critical region.[7] Most commonly, the small device by virtue of its lightness, is used to extend the high-frequency response of the system. Such a high-frequency helping device would be useful in conjunction with a <u>single tank system</u> for which Ω_{st} was infinite or imaginary, because such a system could not by itself produce appreciable stabilization beyond about 2.5 times the natural frequency of the ship. This case is not especially interesting, as it does not appear practical to build such single tank systems.

In practical single tank systems, Ω_{st} will almost cer-tainly be positive. For such a system, <u>kinetic reaction</u> actually provides stabilization in the high-frequency range above secondary resonance (see Figure 14), so that no further

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[7] For a good discussion of helping devices as a logical element in servo systems, see Oppelt, <u>Stetige Regelvorgänge</u>, Chapter III, Sections D and E.
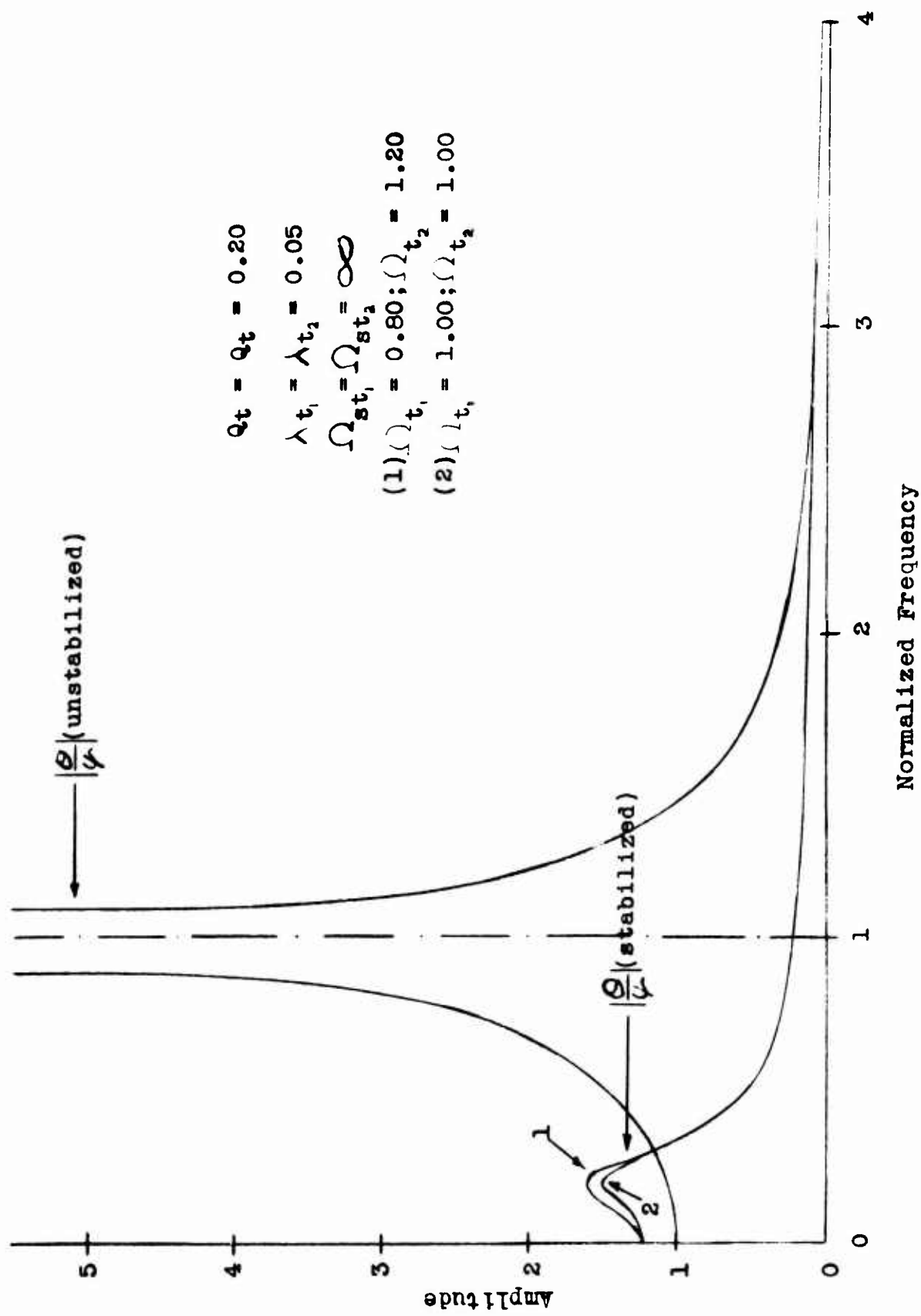
Figure 17. Stabilization with "Stagger - Tuned" diversified tanks.

help is needed at the extreme high frequencies. Help is definitely needed, however, to fill in the secondary resonance null which makes a positive Ω_{st} disadvantageous to a single tank system. We will find that when this null is eliminated by means of a helping tank, the result is a system whose high-frequency performance is markedly superior to that of any possible single tank system.

Mathematics of Double Tanks: In the simple example worked out here, the pump-blade angles of both tanks are positioned by an acceleration signal, but with different control gains. This results in the following block diagram:

$$\Theta = \Theta_{in} - \Theta_{out}$$

$$\Theta_{out} = \Theta_{out_1} + \Theta_{out_2}$$
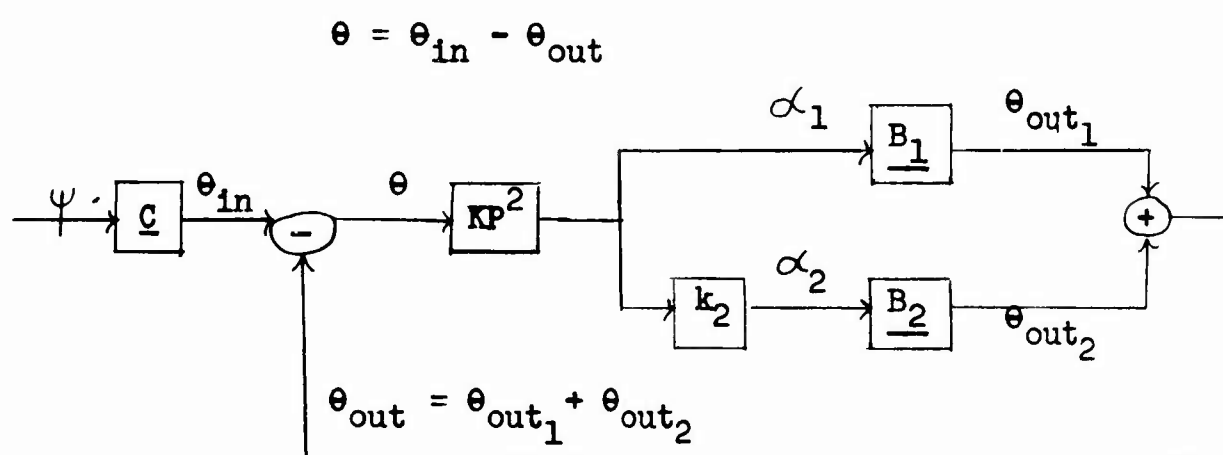
BLOCK DIAGRAM FOR DOUBLE TANKS

From this block diagram the loop-transfer characteristic is,

$$K\,\underline{G} = K\,P^2\,(\underline{B}_1 + k_2\underline{B}_2) = K\,(\underline{G}_1 + k_2\underline{G}_2)$$

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The transfer-responses $\underline{B}_1$, $\underline{B}_2$, and $\underline{C}$ may be found as before from the differential equations of motion. Using an abbreviated notation <u>strictly analogous</u> to that defined for the <u>single tank system</u>,[8] these equations of motion are,

Ship: $[\text{SHIP}]\theta + \lambda_{t_1}\,[\text{ST}_1]\varphi_1 + \lambda_{t_2}\,[\text{ST}_2]\varphi_2 = \lambda_{ss}\,\psi$

Tank 1: $\lambda_{t_1}[\text{ST}_1]\theta + \lambda_{t_1}\,[\text{TANK}_1]\varphi_1 \qquad\qquad = \lambda_{p_1}\alpha_1$

Tank 2: $\lambda_{t_2}[\text{ST}_2]\theta \qquad\qquad +\lambda_{t_2}\,[\text{TANK}_2]\varphi_2 = \lambda_{p_2}\alpha_2$

By Cramer's Rule it follows that,

$$\underline{B}_1 = \frac{\theta_{\text{out}_1}}{\alpha_1} = \frac{\lambda_{p_1}\,[\text{ST}_1]}{[\text{SHIP}][\text{TANK}_1] - \lambda_{t_1}[\text{ST}_1]^2 - \lambda_{t_2}\frac{[\text{TANK}_1]}{[\text{TANK}_2]}\,[\text{ST}_2]^2}$$

$$\underline{B}_2 = \frac{\theta_{\text{out}_2}}{\alpha_2} = \frac{\lambda_{p_2}\,[\text{ST}_2]}{[\text{SHIP}][\text{TANK}_2] - \lambda_{t_2}[\text{ST}_2]^2 - \lambda_{t_1}\frac{[\text{TANK}_2]}{[\text{TANK}_1]}\,[\text{ST}_1]^2}$$

$$\underline{C} = \frac{\theta_{\text{in}}}{\psi} = \frac{\lambda_{ss}\,[\text{TANK}_1]}{[\text{SHIP}][\text{TANK}_1] - \lambda_{t_1}[\text{ST}_1]^2 - \lambda_{t_2}\frac{[\text{TANK}_1]}{[\text{TANK}_2]}\,[\text{ST}_2]^2}$$

And using our standard procedure, we define $\lambda_{ss}=\lambda_{p_1}=\lambda_{p_2}=1$

<u>Results for Single Tank with Helping Tank</u>: The Nyquist diagrams for $\underline{G}_1$ and $\underline{G}_2$ are shown in Figure 18. Notice that

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[8] See Chapter II, Section A of this part.

the helping tank has a higher Q_t, giving it a bandpass action so that it is effective principally in the vicinity of the frequency of secondary resonance. Paradoxically, the locus for the helping tank appears larger than the locus for the principal tank. This is due to the assumption above ($\lambda_{p_2} = 1$), and things are restored to their proper proportion by the choice of $k_2 = 0.2$. Figure 19 shows the $\underline{G}_1$ locus (passing through the origin at secondary resonance), and then the effect of adding the helping tank action. Notice that the effect of the helping tank is to fill in the null and shift the whole high-frequency end of the locus into the right hand plane. This is an ideal form for the locus. It means that so long as the controls can shift the pump-blade angle without lag (i.e. remain ideal), the system will stabilize! The reason for this is that we are no longer using only the displacement of the water to produce counter-torques. Above secondary resonance the stabilizing torques are derived from the inertia reaction, or <u>kinetic reaction</u> of the water.

Thus, with reverse irony, the phenomenon of secondary resonance, which before seemed to be only detrimental, now appears as a useful tool in stabilization by tanks. A stabilizer making use of this principle can produce torques just as fast as the blade angle can be positioned. Its speed of response is <u>on a par or better</u> with fins or any other known stabilizing device. This interpretation and use of secondary resonance is one of the important results of the research at Stanford.

G - plane

Locus of $\underline{G}_2 = P^2 \times \underline{B}_2$

$Q_{t_1} = 0.20; Q_{t_2} = 1.00$

$\lambda_{t_1} = 0.10; \lambda_{t_2} = 0.03$

$\Omega_{t_1} = 1.00; \Omega_{t_2} = 3.00$

$\Omega_{st_1} = 2.00; \Omega_{st_2} = \infty$

Locus of $\underline{G}_1 = P^2 \times \underline{B}_1$

+180°
−180°

5   10   15   20   25   0°
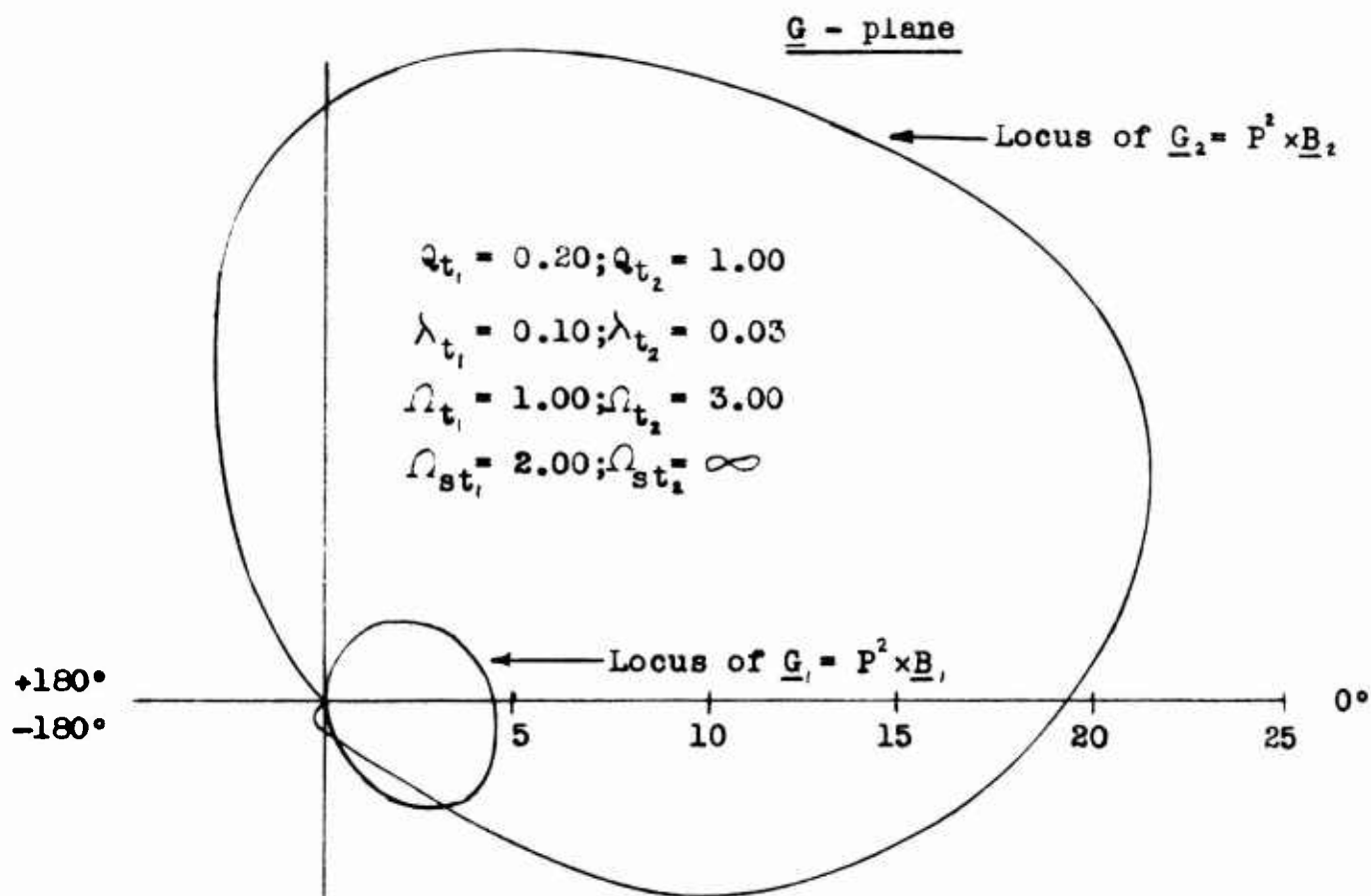
Figure 18. Nyquist Diagrams for the main tank and the helping
tank in a diversified tank system.

G - plane

(1) $\underline{G} = P^2 \times (\underline{B}_1 + 0.2\underline{B}_2)$

(2) $\underline{G} = P^2 \times \underline{B}_1$

Low Freq.
+180°
−180°
High Freq.

.1   .2   .3   .4   0°
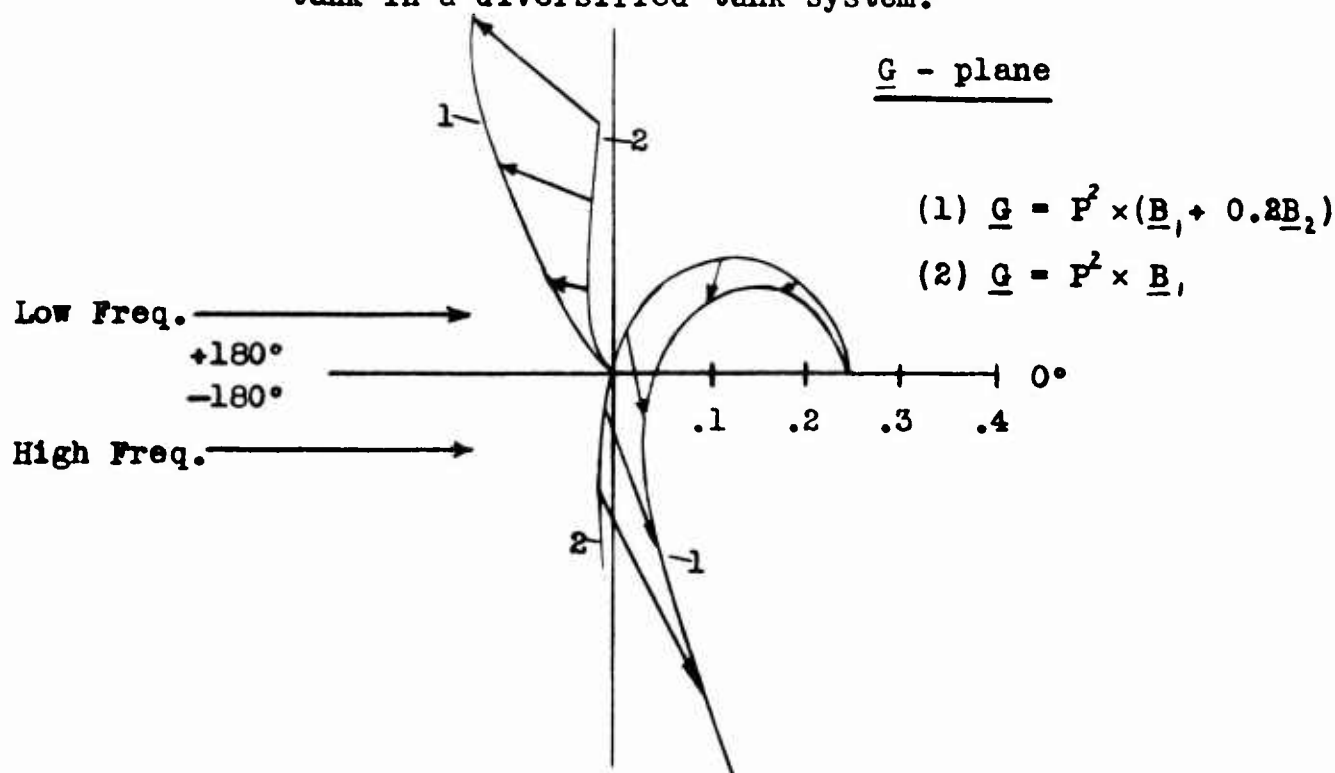
Figure 19. Nyquist Diagram, showing how the helping tank shifts

the overall $\underline{G}$ locus into the right hand plane at high

frequencies.

The full effect of improving the high-frequency end of the locus cannot be realized, until an equivalent improvement is undertaken at the low-frequency end of the locus, as the low-frequency response now limits the gain. This improvement must be done by the use of further controls and hence is discussed in the next chapter. A stabilization curve will not be drawn here for this case, but will be shown when the low-frequency response is improved.

D. Summary

Single Tank System: The results of the study of the single tank system may be summarized as follows. There are four critical parameters: Q_t, λ_t, Ω_t, and Ω_{st}. Q_t should be as low as possible, say about 0.2 - 0.1. λ_t can be as low as practical limitations permit, probably not less than 0.10. Ω_t should be close to 1.0, if anything a little above, say 1.30. Ω_{st} should be as high as it is practical to make it, approaching something like 3.0.

Note that a single tank system with "secondary resonance" at a finite frequency actually stabilizes the high frequencies better than one with "secondary resonance" at infinity. This is an important observation, for it makes it at least plausible that this secondary resonance effect, though disadvantageous in a single tank system, might actually be advantageous in some diversified tank system. And this we found to be the case.

Diversified Tank Systems: Two diversified tanks systems
have been examined. Because of the already low Q_t of the best
single tank system, "stagger-tuning" proved to be of little
advantage. The gain in effectiveness certainly would not
justify the added complexity. On the other hand, significant
performance improvement may be achieved using a single tank
with secondary resonance null filled in by a helping tank. In
this case the theoretical high-frequency loop-transfer locus
was made to lie wholly within the right hand plane. This indi-
cates that the system has a superior high-frequency response.
In physical terms, _inertia reaction_ in the main tank takes
care of the very-high-frequency region, _the helping tank_ takes
care of the secondary resonance null, and _displaced water_ in
the main tank takes care of frequencies below the secondary
resonance region.

Notice that direct acceleration control was used on the
helping tank. The principal locus could be shifted even fur-
ther to the right by using a more sophisticated control for
the helping tank, but _acceleration only_ control illustrates
the point sufficiently well. Actually, a U-tube tank, while
it does the job, is not necessarily the ideal helping device
in this situation. Some other type of tank or even perhaps
small gyros might better be used as the helping agent.

It is interesting, that the high performance of this
double tank system does not depend on a special principle

which could only apply to U-tube tanks, but on the general
ideas of displacement of mass and acceleration of mass. Work-
ing with these ideas is a very powerful way to approach stabi-
lizer design. Stabilizing devices may be classified and
studied according to the basic nature of the torques they pro-
duce and the energy they store. It is already possible to
visualize systems which might perform the functions of this
double tank system in a simpler and even more efficient manner.
Such studies are to form a part of the next year's work at
Stanford, and so will not be discussed further here.

IV. FREQUENCY-RESPONSE APPROACH TO CONTROL DESIGN

Having considered regulated elements, attention may now
be turned to the design of high-performance regulating ele-
ments. The problem is to find those control elements and
control arrangements which lead to the most effective perfor-
mance. A standard ship-tank arrangement is used for the
majority of cases treated in this chapter, although certain
questions will require that a special tank form be used with
the control system being discussed. The standard ship-tank
arrangement will be a single tank system with Ω_{st} = infinity.

As before, we will use the frequency-response approach
and the Nyquist diagram. We will follow the same step-by-step
process, except that in this case the control block transfer-
response, $\underline{A}$, is to be varied, while the ship-tank responses,
$\underline{B}$ and $\underline{C}$, are held relatively constant.

Four more or less distinct control problems are discussed
in the following sections: (a) the use of position and veloc-
ity to augment the acceleration control; (b) the problem of
control lag; (c) the use of "lead" circuits; and (d) the use
of open-cycle control or "feedahead". Hence this discussion
parallels the pattern established in Part A.

A. Position and Velocity as Complementary Controls

The use of position and velocity signals to augment the primary acceleration control signal has been mentioned in Part A. These signals are in a sense _integrals_ of acceleration and therefore should tend to improve the low-frequency response of the system just as "derivatives" improve the high-frequency response. Further integrals such as _integrated position_ could conceivably be used, but they do not seem justified at this time and are not discussed here.

We will first discuss the use of these augmenting signals on a _single tank system_, and secondly discuss their use on a _diversified tank system_, where they will be found to produce an especially effective performance.

Position Signal Aid on a Single Tank System: Acceleration _only_ control gives no stabilization at all at zero frequency. Hence a logical move, is to add a moderate amount of position signal (into the pump-blade angle control) to correct this inadequacy. It can be shown that both signals should have the same sense, meaning that a positive position signal should add arithmetically to a positive acceleration signal. Thus the control transfer-response, $\underline{A}$, has the following form,

$$\underline{A} = \frac{\alpha}{\theta} = K \ (P^2 + k_2)$$

For steady-state sinusoidal oscillation we will find that the position signal dominates at low frequencies, the

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acceleration signal dominates at moderate and high frequencies, but at some intermediate frequency (namely $\Omega = \sqrt{k_2}$) the two signals <u>exactly cancel</u>! It is not wholly coincidence that this effect, at low frequencies, is symmetrically analogous to the secondary resonance effect at high frequencies. The Nyquist diagrams (see Curve #1) in Figures 20 and 21 show this analogy clearly when compared to the preceding Figures 14 and 15. The stabilized response for this case is shown in Figure 22. The null between position and acceleration control is obviously undesirable, and by analogy to the "helping" tank a "helping" control signal is indicated to fill in this null. The logical control signal for this purpose is <u>velocity</u>.

<u>Position and Velocity Signal Aid on a Single Tank</u>: It can be shown that all these signals should have the same sense. Hence, for <u>A</u> we have,

$$\underline{A} = \frac{\alpha}{\theta} = K \ (P^2 + k_1 P + k_2)$$

Figures 20 and 21 (Curve #2) show how the addition of velocity control shifts the low-frequency end of the loop-transfer locus to the right, exactly as the "helping" tank shifted the high-frequency end of the locus to the right! The stabilized response for this case is shown in Figure 23. Analogously to the double tank with acceleration <u>only</u> control, the maximum gain is limited by the <u>unimproved</u> end of the locus, and so the complete effect of this improvement cannot be realized.

- 28 -
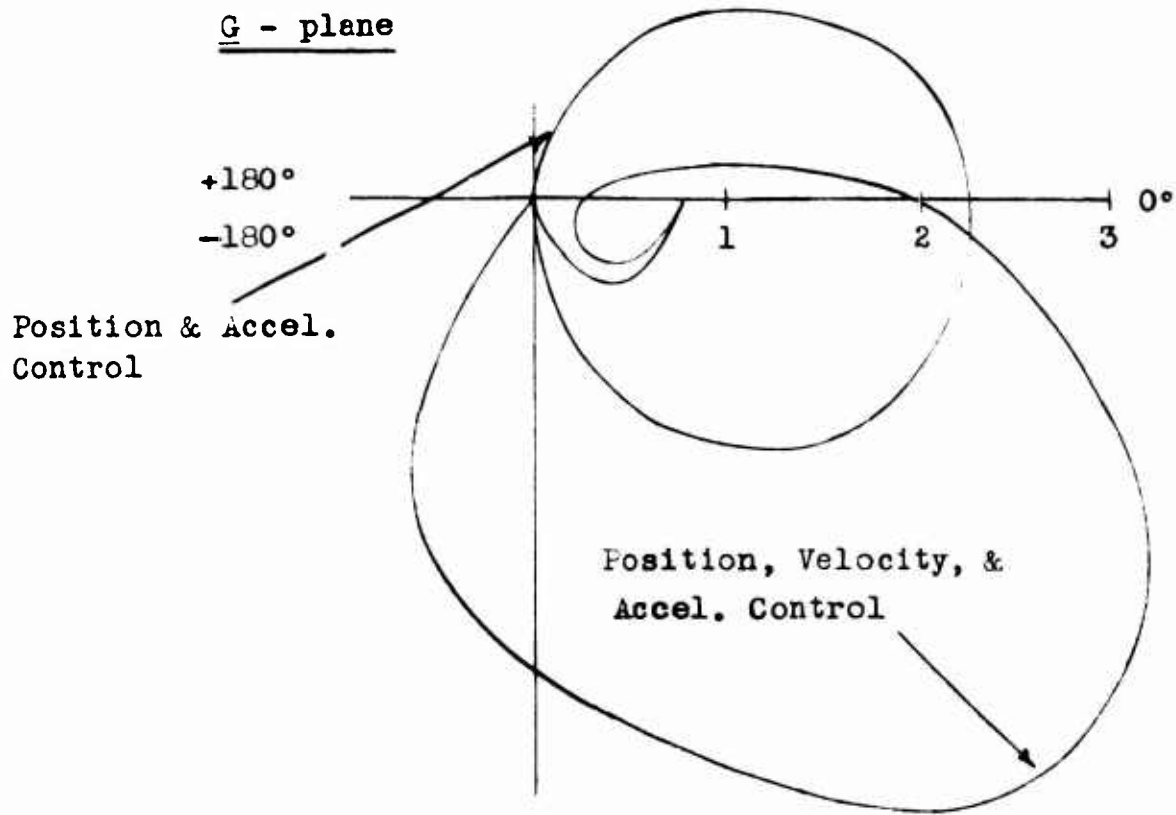
Figure 20. Nyquist Diagram - Exaggerated effect of adding position
and velocity control.
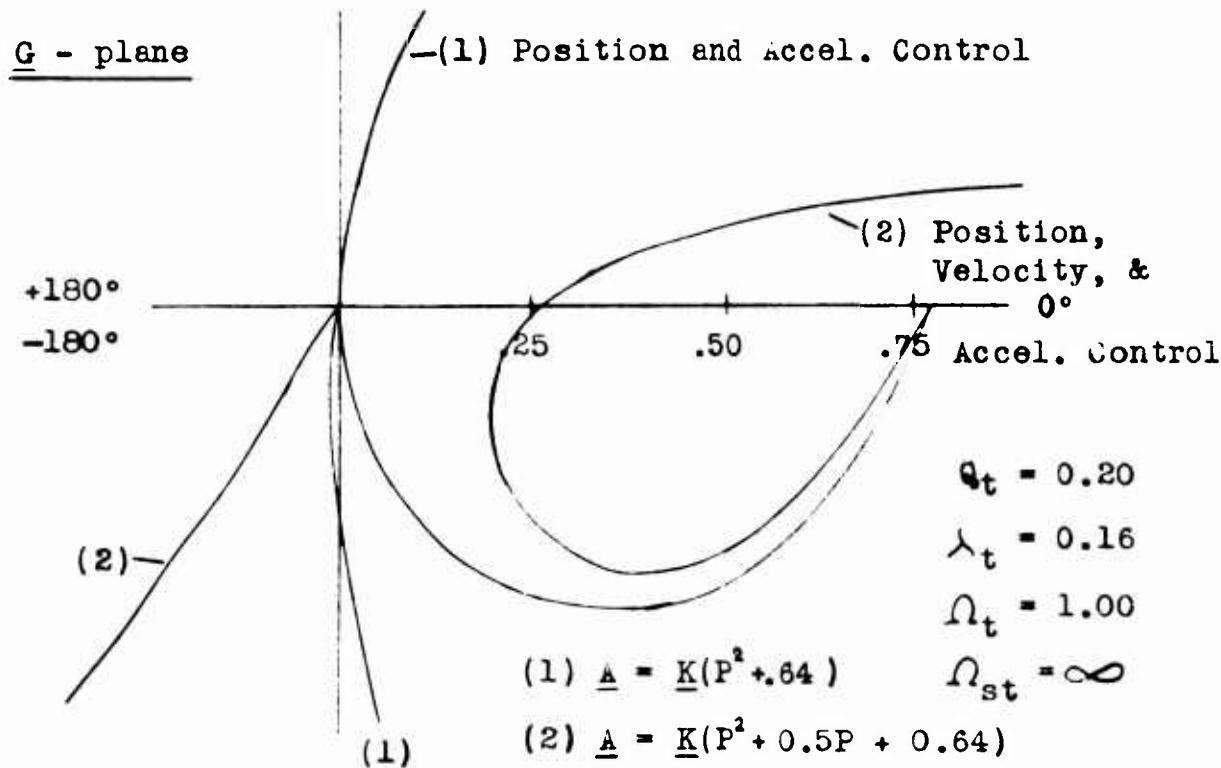
Figure 21. Nyquist Diagram - Actual effect of adding position and
velocity control.

$\theta_t = 0.20$

$\lambda_t = 0.16$

$\Omega_t = 1.00$

$\Omega_{st} = \infty$

(1) $\underline{A} = K \times P^2$

(2) $\underline{A} = K(P^2 + 0.06)$

(3) $\underline{A} = K(P^2 + 0.64)$

$\left|\dfrac{\theta}{\psi}\right|$ (unstabilized)

$\left|\dfrac{\theta}{\psi}\right|$ (stabilized)

Normalized Frequency

Amplitude
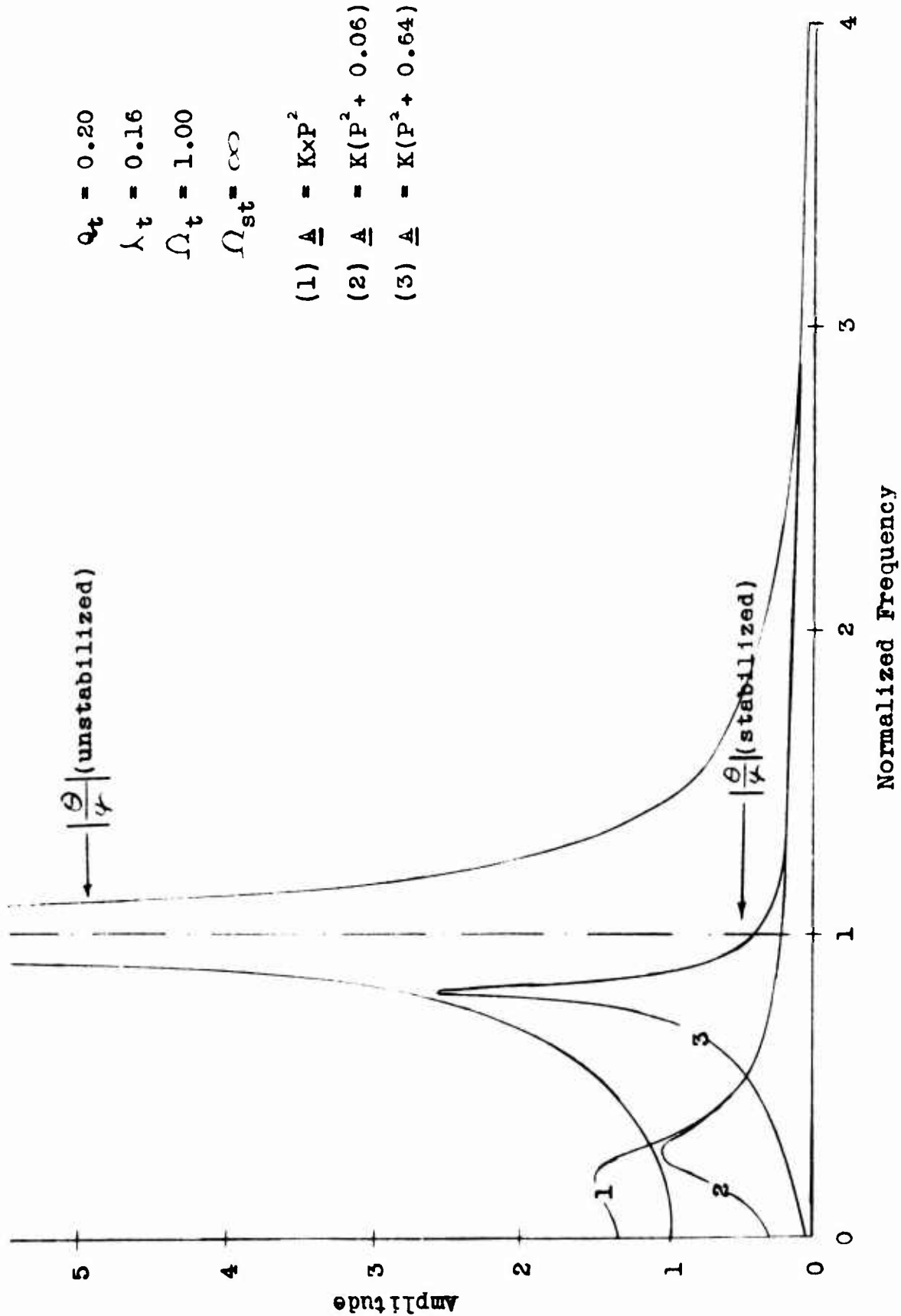
Figure 22. Comparative stabilization with and without position control added.
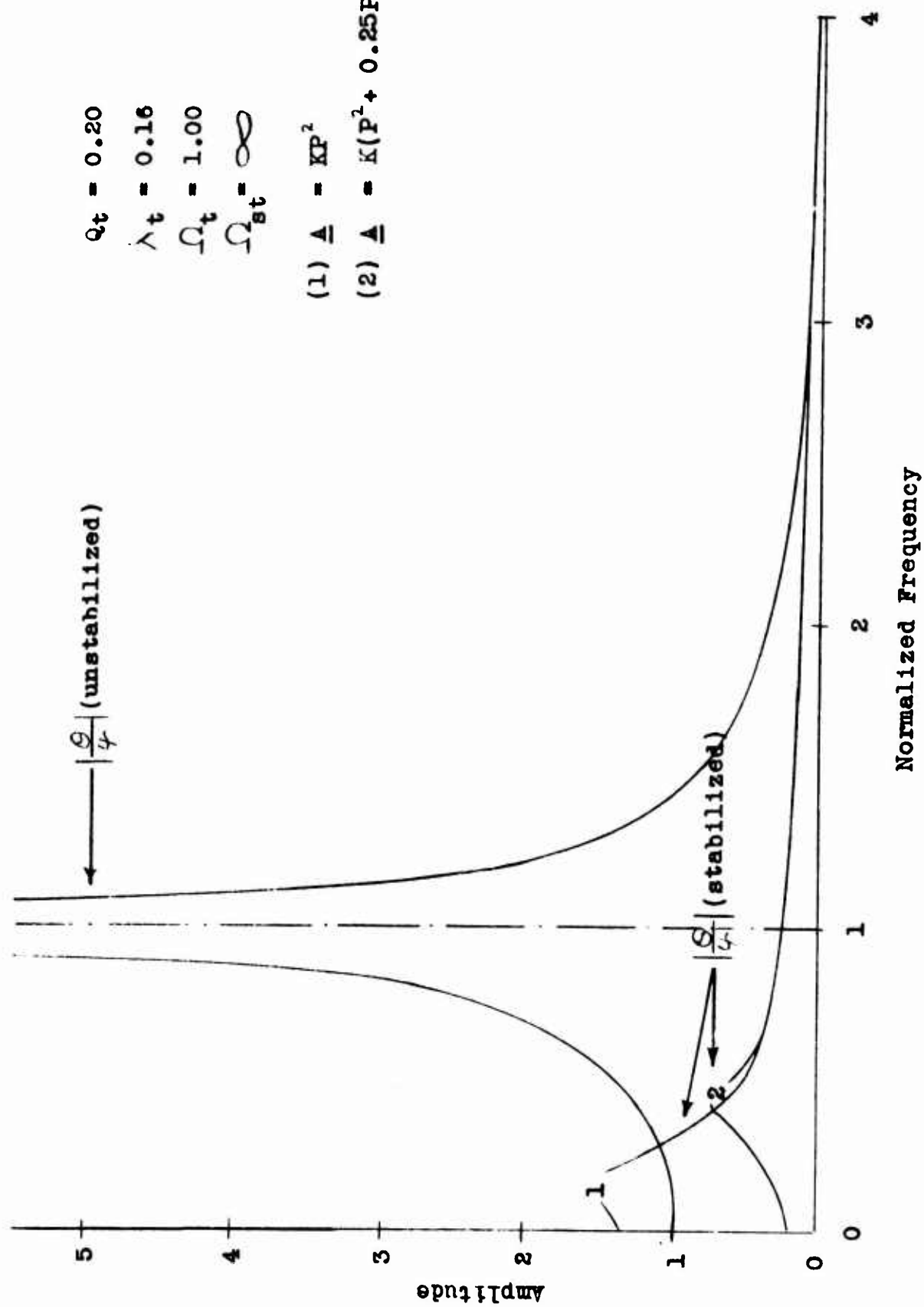
Figure 23. Comparative stabilization with and without both position and velocity control.

In order for maximum effectiveness to be realized, it is necessary to use at one and the same time, the improvement gained at low-frequencies by the added control signals, and the improvement gained at high-frequencies by the helping tank.

Position and Velocity Signal Aid on Diversified Tanks:
In this case we use acceleration plus velocity plus position signal control on the last-discussed double tank system. Thus the loop-transfer locus, K $\underline{G}$, is given by,

$$K \underline{G} \equiv \underline{A} \times \underline{B} = K (P^2 + k_1 P + k_2) (\underline{B}_1 + 0.2 \underline{B}_2)$$

This loop-transfer locus is plotted in Figures 24, 25, and 26. It is seen that the locus now lies completely in the right hand plane, a most desirable state of affairs. With this locus it would be possible, in fact, to make the loop gain infinite, and hence the stabilization perfect, without exceeding the magnification ratio of 1.3. In practice, however, the completely ideal controls assumed here are not realizable. The high-frequency response will eventually drop off due to deficiencies in the control, i.e. due to the inability of the controls to shift the pump-blade angle without lag at very high frequencies. The important point to note, is that the burden of lag has been transferred from the relatively unwieldly regulated elements to the more flexible and versatile controls!

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G - plane

Small Scale

Low Freq. ————→

+180°

−180°

High Freq. ————→

1 2 3 4 5 6 7   0°
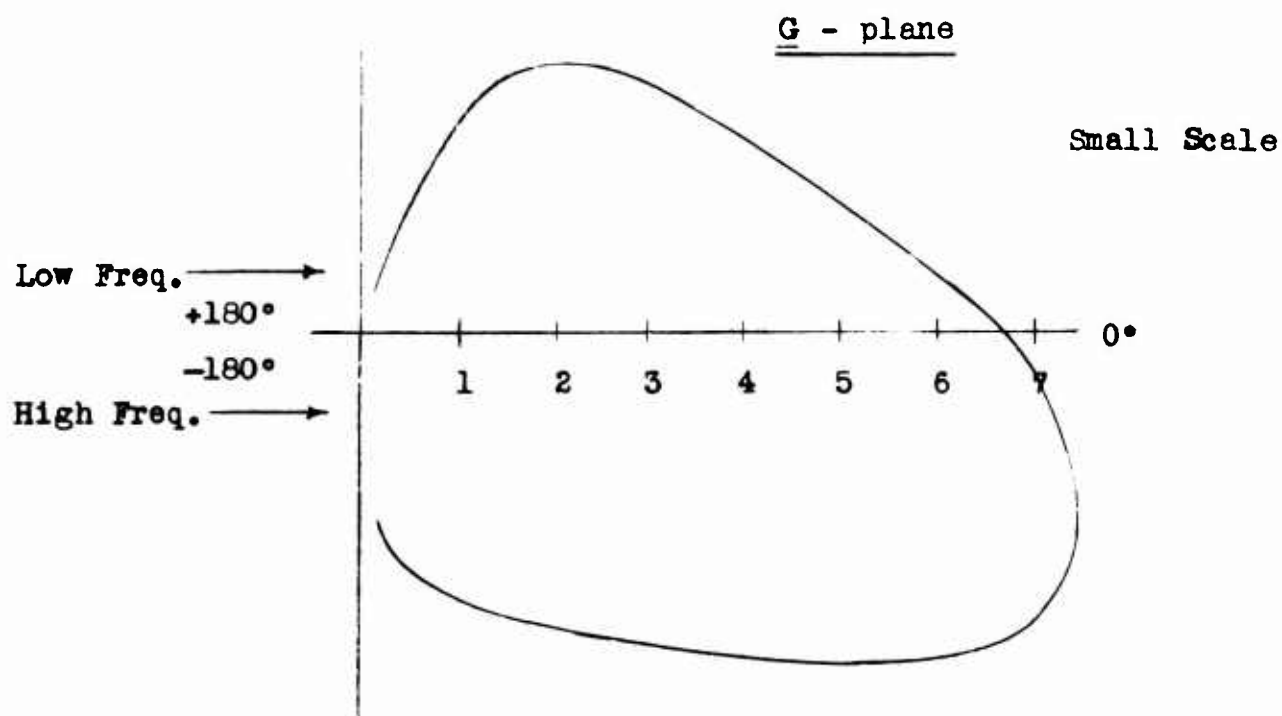
Figure 24. Nyquist Diagram - G locus for a double tank with position

and velocity control added.

G - plane

Large Scale

Low Freq. ————→

+180°

−180°

High Freq. ————→

.1   .2   .3   0°
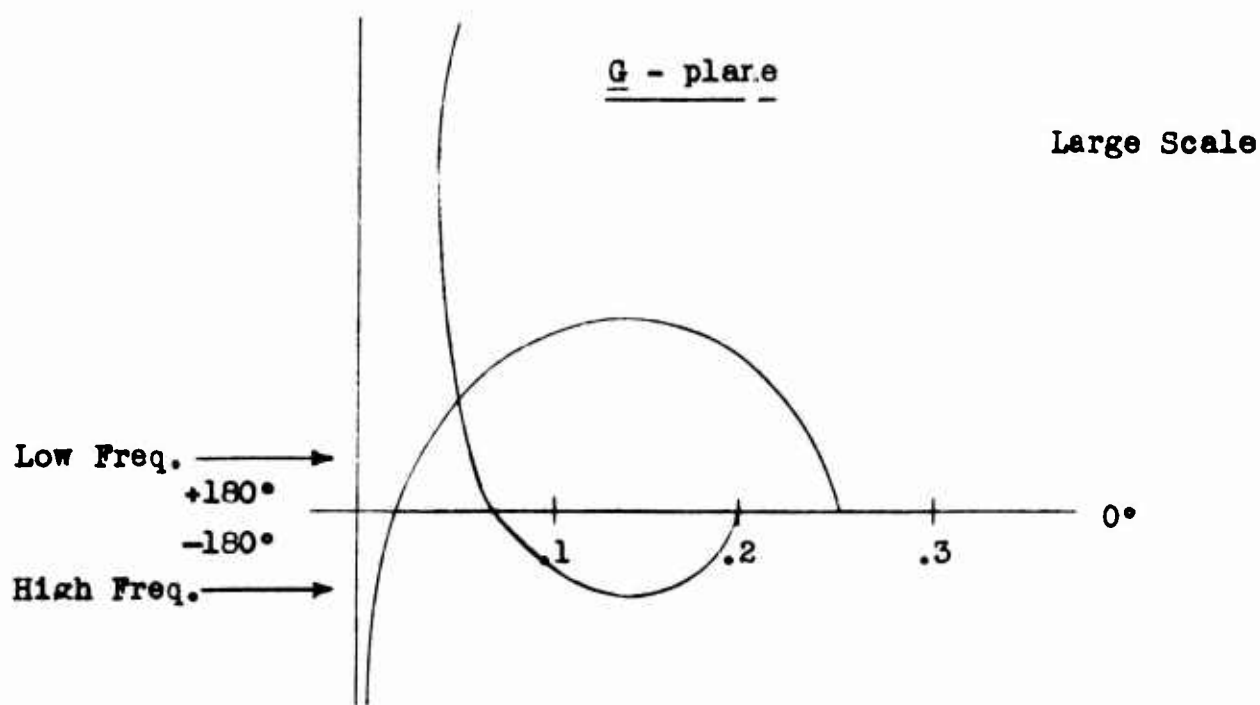
Figure 25. Nyquist Diagram - G locus for double tank with position

and velocity control added.

At low frequencies the controls <u>can</u> be considered ideal without violence to the physical facts, so this end of the locus is reliable. While the control action must eventually fall off as frequency is increased, the critical frequencies of the control system should be well beyond the ship's natural frequency, perhaps 5-10 times the ship's natural frequency. As discussed in Part A, the high-frequency control response problem does not seem excessively difficult in the case of ship stabilization.

A locus such as that in Figure 26 does not provide a magnification ratio criterion by which to set the gain. The gain could be set analytically, only by further specification of the <u>non-idealness</u> of the control system. Such specification lies beyond the scope of this report. The gain has <u>arbitrarily</u> <u>been assumed</u> to be <u>four</u> times the maximum allowable gain for the <u>single tank system</u>, which seems to be a reasonable engineering estimate. The somewhat startling result is shown in Figure 27. If one remembers that most of the rolling of the ship is resonance rolling, and so larger than the waveslope, the stabilization curve in Figure 27 is seen to represent something in the order of 95% reduction of "average" rolling. This means that for maximum effective waveslopes of 6-10 degrees, the ship's roll would be held to something like one degree, assuming, of course, that the <u>capacity</u> of the system was not exceeded.

Single Tank

$q_t = 0.20$
$\lambda_t = 0.16$
$\Omega_t = 1.00$
$\Omega_{st} = \infty$

(1) Single Tank, Accel. Control

(2) Double Tank, Position, Velocity, and Accel. Control

$\left|\dfrac{\Theta}{\psi}\right|$ (unstabilized)

$\left|\dfrac{\Theta}{\psi}\right|$ (stabilized)

Normalized Frequency

Amplitude
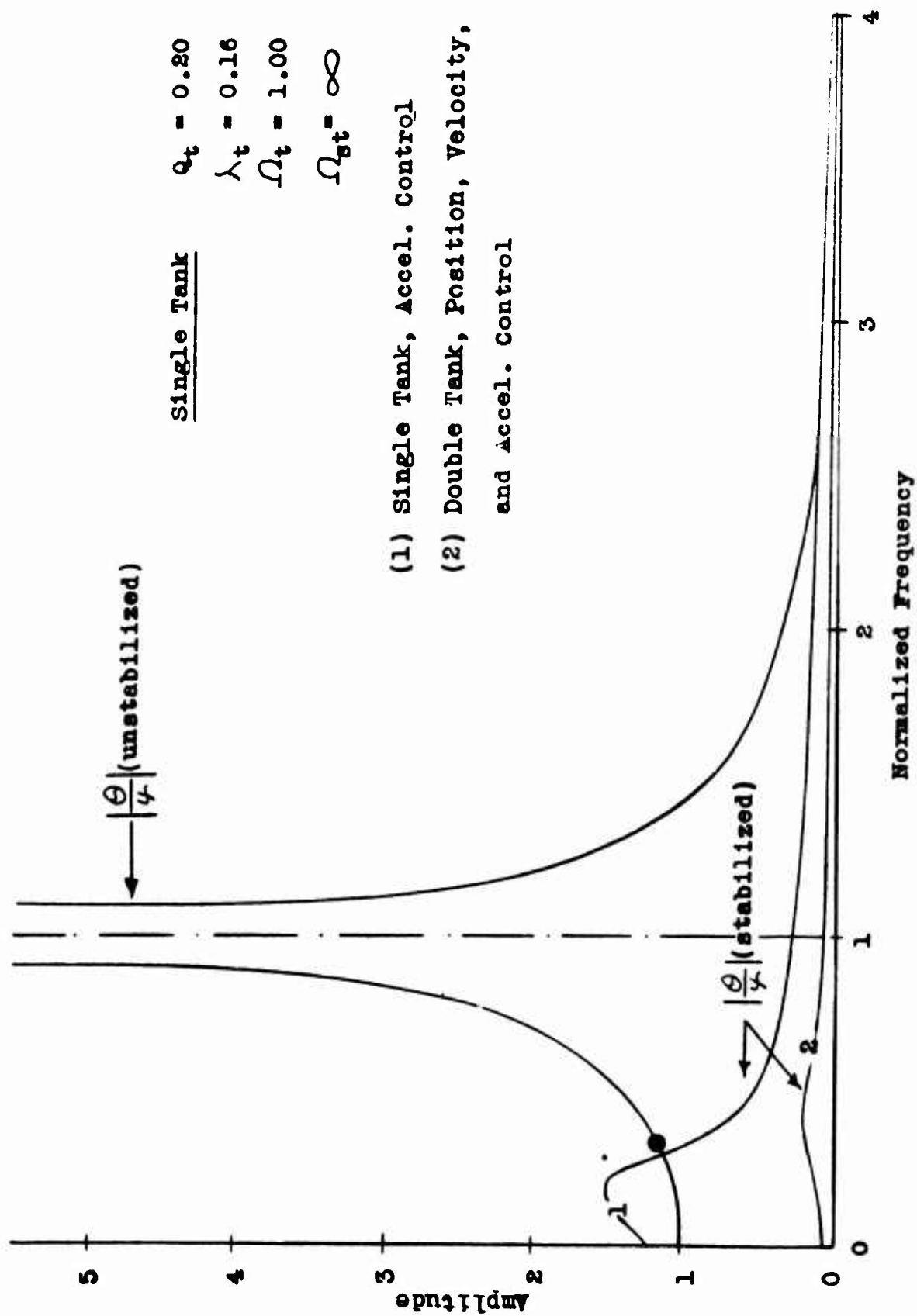
Figure 27. Comparative stabilization for the double tank with position and velocity control added.

With this last result, the assumption of ideal controls has clearly reached the limit of its usefulness. It would be unprofitable to carry the quantitative treatments in this chapter further toward the optimum without a more specific definition of the controls. As mentioned above, such specification, with a consequent loss of generality, lies outside the proper scope of this report.

Let us review what has been accomplished up to this point. By judicious use of tanks and controls the all-important loop-transfer response, $K \underline{G}(j\Omega)$, has been converted from a narrow "bandpass" response (for the single tank system with high Q_t) to an "allpass" response (for diversified tanks with aided control). That is, the loop-transfer response is "allpass" for ideal controls. It would naturally be a "lowpass" response in the case of non-ideal controls, but a lowpass response whose cutoff is wholly in the controls, and not in the regulated elements! This "lowpass" problem resembles the usual servo problem and its treatment in terms of frequency-response methods is straightforward.

Having now designed a locus whose cutoff depends only on control lags it is fitting to illustrate what can happen if the controls are allowed to introduce excessive lag into the system. It should be noted that, in principle, it seems possible to build quite adequate controls for this system, but that in practice it is not at all difficult to build controls with very unsatisfactory characteristics.

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B. Control Lags: Their Origin and Their Effect

As this project has not included work on the actual development of optimum controls, what is said here is intended to be of a qualitative and general nature, and to represent what may reasonably be inferred from the known state of the control and instrumentation art.

For ship stabilization, the assumption of ideal controls is sufficiently valid over the low and middle frequencies. The approximation will inevitably break down as the frequency becomes relatively high. As the frequency becomes higher and higher, the pump-blade angle will first lag behind the control signals and eventually fail to follow them altogether. Thus we speak of high-frequency lags, and of high-frequency "cutoff". Let us consider where, why, and when, such lag will occur; and what the general effect on performance will be.

Three Sub-Blocks in the Control System: As the control block has been defined in this report it includes all elements interposed between the ship's angle of roll and the pump-blade angle. This control block can be broken into three sub-blocks, each with a different function and nature: (1) the sensitive elements or detecting elements; (2) the control amplifier or computing element; and (3) the positioning motor or output element. The functional relationship of these control blocks is shown in Figure 28a.

- 32 -
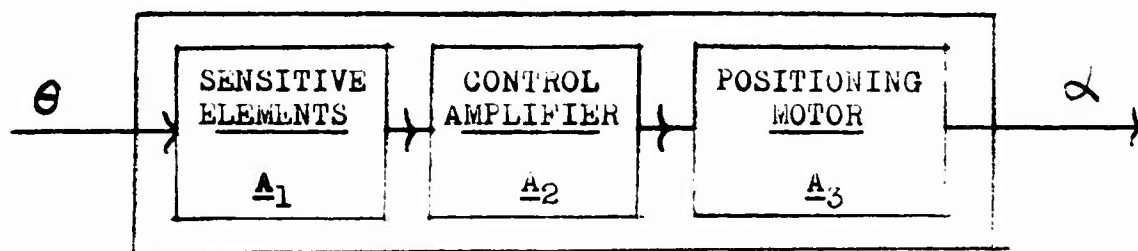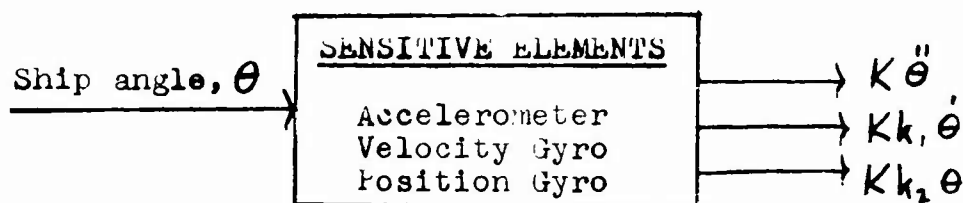
Control Block, A

Sub-Block, A_1
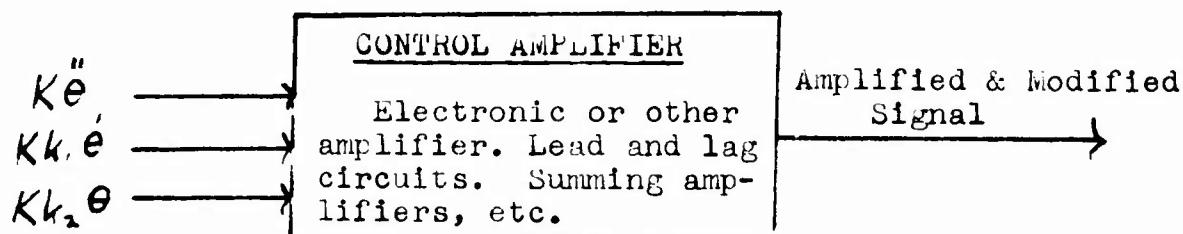
Sub-Block, A_2
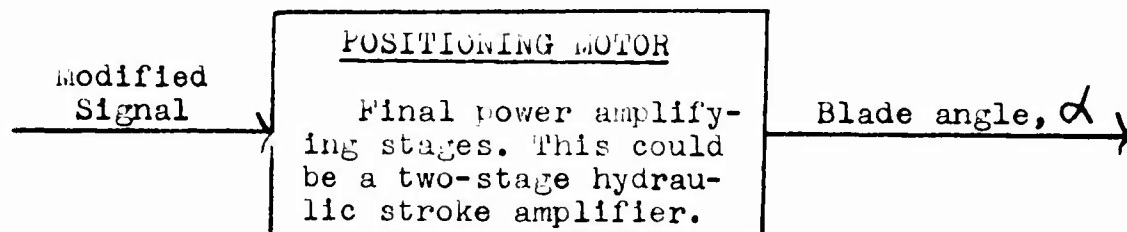
Sub-Block, A_3

Figure 28a.   Definition of Sub-Blocks in the Control

<u>Lag in the Sensitive Elements</u>: A control action based on acceleration plus velocity and position signals has been described in the preceding paragraphs. These signals can best be obtained by direct measurement. The three instruments required, would in all probability be an angular accelerometer, a velocity gyroscope, and a position gyroscope -- at least these will serve for the argument.

All of these instruments tend to give ideal (no lag) response as frequency approaches zero, but to lag at high frequencies. By the nature of the control action, however, <u>it is not necessary</u> that the position and velocity signals "hold up" into the high-frequency range. Clearly, the position and velocity signals need only "hold up" until the accelerometer signal has become the predominant signal. As the accelerometer should be the predominant signal at the ship's natural frequency and above, the position and velocity signals are only required to approximate the ideal in the range of frequencies 0 - 0.15 cycles/second. This is not a severe requirement, hence the position and velocity instruments can be considered ideal to all intents and purposes, insofar as their frequency response is concerned.

The accelerometer, which provides the predominant control signal above the ship's own frequency, should have a flat response well into the high-frequency range. Considering the present state of the art of accelerometer design, this again is not a severe requirement, and the accelerometer can be considered lag-free.

Lag in the Control Amplifier: If an electronic amplifier is used as in the model system at Stanford, its response can be considered lag-free. "Differentiators" cannot, of course, produce ideal derivatives, but the less ambitious "lead" and "lag" circuits can generally be expected to give their designed response over the desired frequency range.[9] Whether electronic or otherwise, the elements of the control amplifier will generally be lighter, faster acting, and hence considerably more free from lag, than the elements of the positioning motor which are called upon to expend a fair amount of power.

Lag in the Positioning Motor: As defined here, the positioning motor includes all those elements which take part in converting the output of the control amplifier into the angle of the pump blade. For example, the positioning motor might be a two-stage hydraulic stroke-amplifier, as in the Denny-Brown fin control system. In the model system, the positioning motor consists of a small two-phase servomotor in series with a hydraulic stroke-amplifier. The small servomotor unfortunately has more lag than any other element in the whole control system. This need not be the case in future practice. The principal lag of the control system should be expected to reside in the final power element, i.e. the final blade-angle-positioning device. It further appears that even this lag should not be

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[9] See Brown and Campbell, Principle of Servomechanisms, Chapter 7, for a discussion of differentiators versus lead and lag circuits. This chapter is concerned with the whole problem of G function synthesis.

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serious until one reaches frequencies of 5-10 times the ship's natural frequency.

Fictitious Example of Control Lag: The discussion above seems to indicate that the inherent limitations on control response are not unusually severe in ship stabilization by tanks. One should not underestimate the difficulty of the control design problem, however. It requires not only the best possible components, but a very nice balancing of their sensitivities, ranges, and responses. Each element must be almost tailor-made to fit its particular function, for the old adage "a chain is no stronger than its weakest link" applies here with double emphasis.

Figure 28 shows the very real deterioration in performance due to excessive lag. This is only a slight exaggeration of the conditions existing in the 12-year-old model system at Stanford. For this example, the two-phase servomotor is assumed to be contributing the important lag (as in the model), and to have the following overall response:[10]

$$\frac{\text{out}}{\text{in}} = \frac{1}{1+(\frac{P}{2})+(\frac{P}{2})^2} \quad ; \quad P = j\Omega = j\frac{\omega}{\omega_s}$$
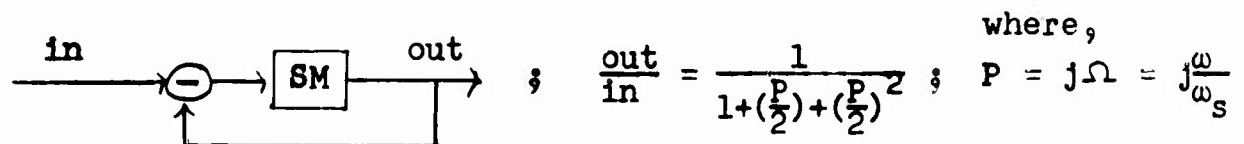
Overall Response of Servomotor

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[10] Servomotors and their response are discussed in any standard servo text. For example see Chapter 2, page 45, in Brown and Campbell, Principles of Servomechanisms.
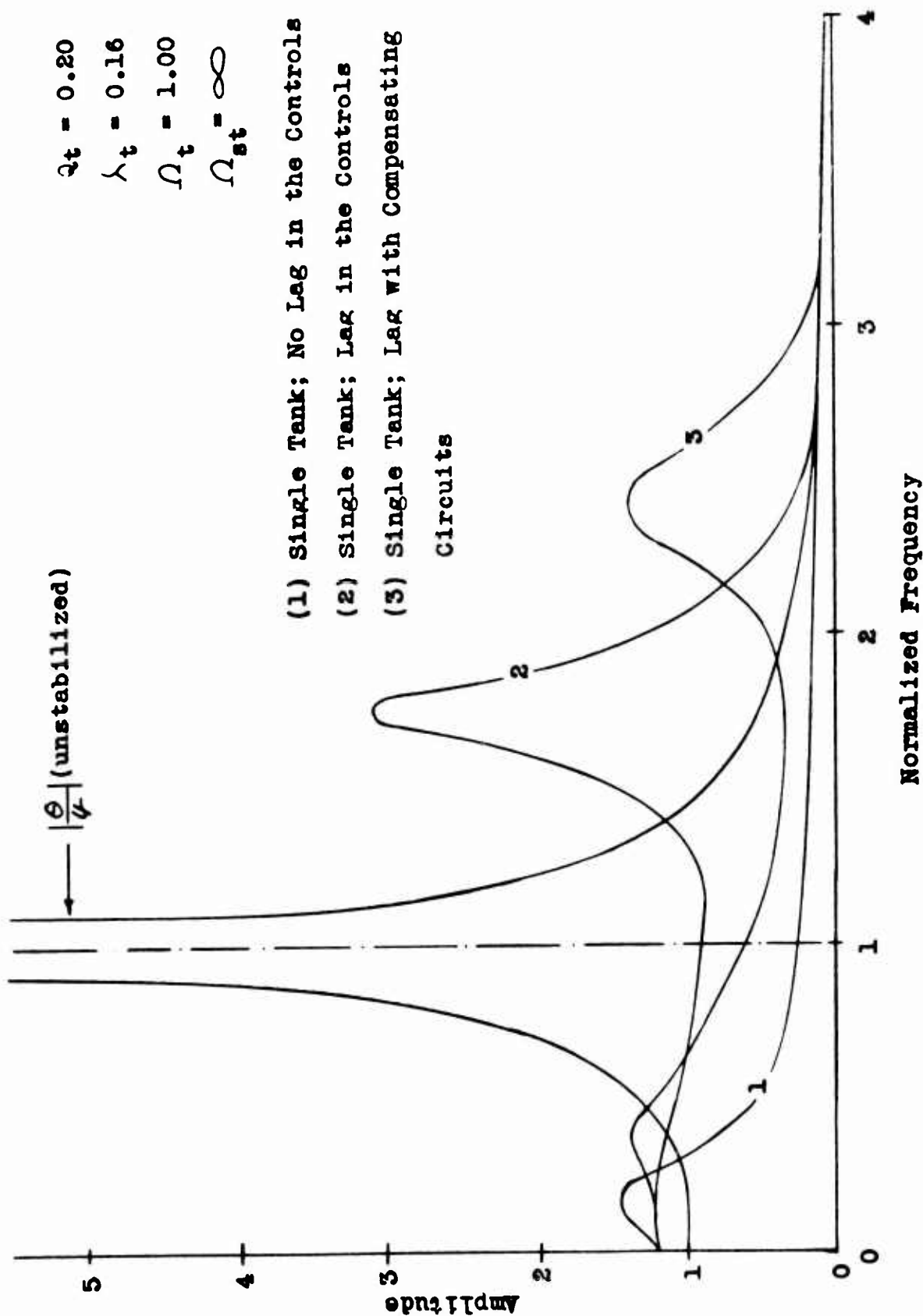
Figure 28. Stabilization with lags in the control system; Uncompensated and Compensated.

This means that the servomotor has a frequency response similar to that of a pendulum whose natural frequency is twice that of the ship and whose damping is one-half critical.

C. Uses of Lead Circuits

Lead circuits are the <u>physically realizable</u> counterpart of the <u>non-realizable</u> differentiator. Their function is to provide phase lead in the control action, and while they cannot provide pure derivatives, they may provide the equivalent of derivative control over a certain range of frequencies. They and their opposites, the lag circuits, are often thought of as devices by which one may shape the loop-transfer locus into more desirable form. They are treated extensively in the literature. As we have made reference to lead circuits a number of times, it may be worthwhile to illustrate several ways in which they might be employed in this problem.

<u>Lead Circuits to Reduce the Effect of Lag</u>: Lead circuits are often used to reduce the effect of high frequency lag such as was discussed in the preceding section. Curve #3 in Figure 28 shows the use of a simple series lead circuit to reduce the effect of the gross lag in the two-phase servomotor. Naturally one cannot expect perfection in the presence of such lag, but the improvement is marked. The series lead circuit was used in the following way.
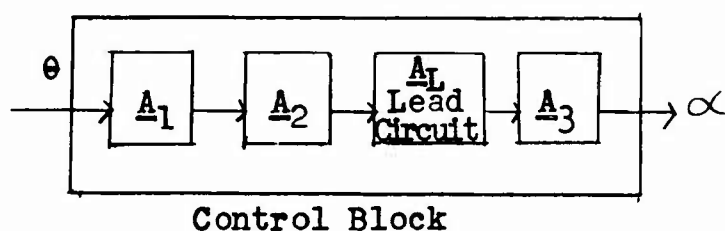
Control Block

$$A_L \equiv \frac{1 + \dfrac{P}{0.67}}{1 + \dfrac{P}{6.00}}$$

Series Lead Circuit Compensating Two-Phase Servomotor

Use of Lead Circuit to Replace Velocity Gyro: A similar
series lead circuit can be used in an interesting way in the
first control sub-block, A_1, to replace the velocity gyro.
Notice that a simple lead circuit tends to generate a mixture
of the input signal and its first derivative. If the denomina-
tor were equal to unity, the circuit would generate the input
plus its first derivative, but the variable term in the denomina-
tor eventually (at high frequencies) nullifies the derivative
action. Even so, a circuit of this type can be used to eliminate
the velocity gyro, so that only position and acceleration sig-
nals need be generated. The block diagram of A_1 would be as
follows,
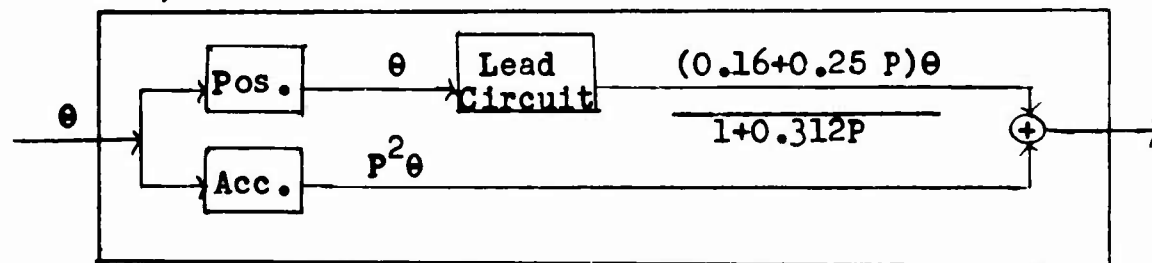
Figure 29a. Use of Lead Circuit to Replace Velocity Gyro

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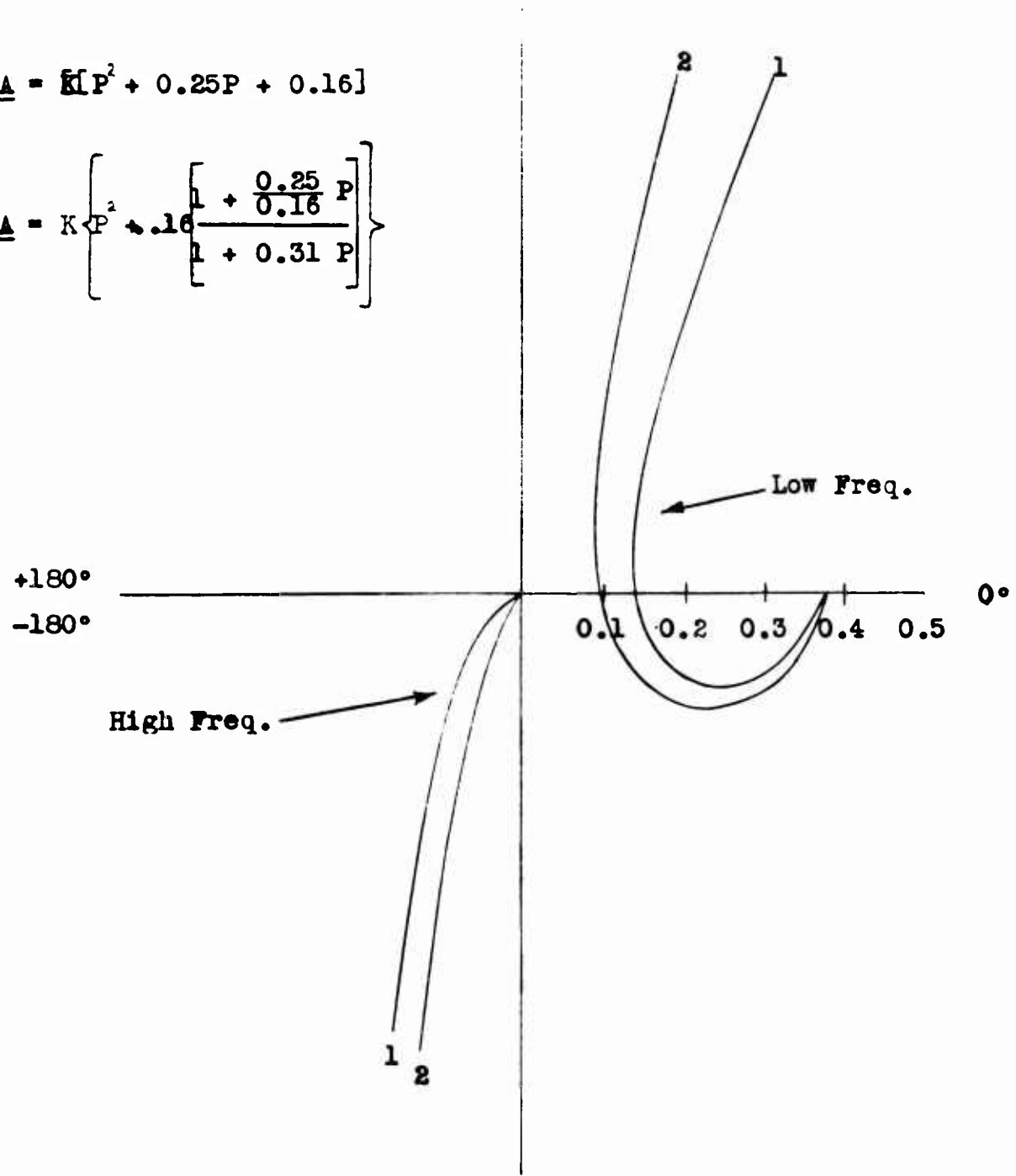
Figure 29. Nyquist Diagram - Use of equalizing circuit to obtain equivalent velocity control.

By the same token that the position and velocity signals need not be ideal beyond the point at which the acceleration signal predominates, the "differentiation" of the lead circuit need not be ideal beyond this same point. Such a circuit could be designed without difficulty. Its use with a single tank system is shown in the Nyquist diagrams of Figure 29, where its action is compared to that of the original acceleration plus velocity and position control.

D. "Feedahead" or Open Loop Control

Up to this point, the discussion of controls has centered around the idea of feedback or so-called closed loop control. Various modifications of the loop-transfer locus leading to more effective performance have been proposed and illustrated. Feedahead, on the other hand, is a system of control with no closed loop and hence no loop-transfer locus. When it can be used it complements the feedback action in an important way. Physically, feedahead is a reversion to direct control, i.e. the kind of control that had to be used before feedback was invented. The advantage of feedahead control lies in the fact that one need not give up the useful feedback control, both can be used simultaneously; and, as shall be seen, their separate effects are multiplied. The servo block diagram with feedahead added is shown in Figure 30. As indicated there, the feedahead control measures the waveslope and then attempts to

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Block diagram for Feedback and Feedahead Control

It follows by simple algebra from the above that,

$$\frac{\theta}{\psi} = \underline{C} \times \left(\frac{1}{1 + \underline{A}\,\underline{B}}\right) \times \left(1 - \frac{\underline{X}}{\underline{CB}^{-1}}\right)$$

$$\frac{\theta}{\psi} = (\text{Frahm stab.}) \times (\text{Feedback stab.}) \times (\text{Feedahead stab.})$$

$$\frac{\theta}{\psi} = \text{Overall stabilization with feedback and feedahead}$$

Now, for the best feedahead action, we desire that,

$$\underline{X} = \underline{C}\,\underline{B}^{-1} \quad \text{so that} \quad \left(1 - \frac{\underline{X}}{\underline{CB}^{-1}}\right) = 0 \quad \text{and} \quad \frac{\theta}{\psi} = 0$$

Figure 30. Feedahead Control and its Effect on
Overall Stabilization

position the blade angle so that this waveslope is exactly counteracted. It never knows how successful it is in this operation for presumably there is no feedback from the ship's motion to the waveslope.

At the same time time that this is going on, the feedback control measures the residual motion of the ship and acts to reduce it. This has led some authors to speak of feedahead and feedback control as a sort of coarse and fine correction system. Whether one chooses to think in such terms or not, it is clear that each action is independent of the other, and that the net effect is the product of both actions. This is an ideal state of affairs. If, for example, the feedback and the feedahead control each reduce the rolling to 20% of the unstabilized value when acting alone, both together would reduce the rolling to 20% x 20% or 4% of the unstabilized value! Before the reader gains the false impression that Utopia has been reached, it may be well to point out and discuss the two critical factors on which the realization of a feedahead system of control hinges: (1) measurement of the input, in this case the effective waveslope; and (2) synthesis of the feedahead control function, $\underline{X}$.

Measurement of the Waveslope, ψ : For feedahead control it is first necessary to measure the effective waveslope, ψ . Furthermore, this measurement must be essentially unaffected by the ship's roll, so that no feedback takes place in this part of the control. Pressure devices have been spoken of in this

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regard[11] and it is clear that they are affected by the wave-slope in much the manner one would like. Unfortunately they are also affected by the ship's angle of roll, so that their signal is roughly a measure of $\psi - \theta$. This cannot be used directly, as the ship's roll angle component in the signal would close the loop. It is not inconceivable, however, that the ship's roll angle could be measured elsewhere and subtracted out of this signal leaving a "pure" waveslope signal. Such a signal would be very useful not only for operational ship stabilization but for oceanographic studies concerned with effective waveslopes. It is not possible to estimate just what can be achieved along these lines at present, but the matter certainly deserves further consideration.

Synthesis of the Feedahead Control Function, X: Once the waveslope has been measured, the effectiveness of feedahead depends on the degree to which $\underline{X}$ can be made to approximate $\underline{C}\ \underline{B}^{-1}$, and the frequency range over which this approximation holds good. It is perfectly possible that the two may be exactly equal at one or more points, making the ship totally stabilized against excitation at such frequencies. In general, however, $\underline{C}\ \underline{B}^{-1}$ is not likely to be realizable, so that $\underline{X}$ cannot equal it over the whole range, even in principle.

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[11] Principally by T. A. Roccard, "Stabilizing Equipment for Vehicles, Particularly Ships," U. S. Patent 2,130,929, Sept. 20, 1938.

The problem resolves itself into a rather tricky approximation problem, which depends on the nature of the $\underline{B}$ and $\underline{C}$ in question. One should try to design $\underline{X}$ in such a way that it falls off at high frequencies faster than $\underline{C}\ \underline{B}^{-1}$ so that as the feedahead ceases to stabilize it does not destabilize. It seems quite possible to design $\underline{X}$ in this way. Again the details of design can only be worked out in terms of situations more specific than we are treating in this report. These questions will be studied in the further work at Stanford.

E. Summary

Position and Velocity as Complementary Controls: We investigated the use of position and velocity signals to augment the basic acceleration control signal. The result was an improvement in the low-frequency response completely analogous to the improvement in the high-frequency response, due to the use of secondary resonance and a "helping tank". When position and velocity aided control were used on a diversified tank system of this last-mentioned kind, the whole loop-transfer locus was moved into the right hand plane, producing a highly effective stabilization action. Further improvement was then seen to be a function of the non-idealness or lag of the controls.

Control Lags: Their Origin and Their Effect: The control block may be divided into three sub-blocks: (1) sensitive

elements; (2) control amplifier; and (3) positioning motor.
The lag in the first two of these should be nearly negligible.
Because of its size and weight, the final positioning element
may be expected to contribute the greatest part of the lag in
the control system. It appears possible to operate the controls
out to frequencies of 5 - 10 times the ship's natural frequency.
However, if lag exists, for one reason or another, its effect
can be very serious!

Uses of Lead Circuits: Lead circuits are the physically
realizable counterpart of differentiators. Instead of acting
as "pure" differentiators, they produce an input plus deriva-
tive signal over a certain range of frequencies. They may be
used to reduce the effect of system lags. This was shown. In
certain instances they may be used to replace differentiators.
This was illustrated by using a lead circuit to replace the
velocity gyro.

Feedahead or Open Loop Control: The closed loop or feed-
back may be assisted by open loop or feedahead control. From
the block diagram the two actions were seen to be independent,
and to give a final stabilization equal to the product of the
two separate stabilizing actions. Feedahead also gives the
possibility of total stabilization (at least at certain fre-
quencies), which feedback does not give. The practical use
of feedahead is dependent on being able to measure the effec-
tive waveslope, ψ, independent of the ship's motion. This,

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of course, cannot be done by pressure detectors alone. We also must synthesize the feedahead control function, $\underline{X}$. In addition to being useful operationally, a signal corresponding to the effective waveslope, ψ, would be very useful for oceanographic purposes.

V. THE USE OF WAVESLOPE INFORMATION IN DESIGN

A. Introduction

The purpose of this chapter is to explore in a brief and general way the relation between our knowledge of waveslopes (the input) and effective design. In doing this we are looking toward the future more than in the preceding chapters, for our knowledge of waveslopes is far from complete, and the techniques by which we may make use of this knowledge are not yet fully developed. On the other hand, the input in the ship stabilization problem is especially interesting; and it is not impossible, in the author's estimation, that some day ship stabilization may come to be looked upon as a classic problem in this regard.

The usefulness of the frequency-response approach is self-evident in the two preceding chapters; as a first approach it is perhaps unexcelled. However, the frequency-response approach does not take the nature of the input into account in any explicit way. Essentially it assumes a unit amplitude input at all frequencies. In practice, this assumption can be tempered by the judgment of the engineer, but there is still no quantitative way in which the frequency-response method can take account of the input nature. We must look for other ways to make use of input information, but before we do this, let us inquire into the mathematical nature of the input.

B. Mathematical Nature of the Input

The ways in which we may make use of input information depend on the mathematical nature of the input. The problem is basically a problem in communications engineering, and the waveslope input may be characterized as a time series. As do many other time series, the effective waveslopes on the ship have two fundamentally different types of qualities: (1) systematic or analytic qualities; and (2) random qualities.

Systematic Qualities: By systematic qualities we shall mean those qualities which follow from a strong cause-effect relationship which can be understood; and from which, seeing the cause, we may predict the effect with reasonable accuracy. As the most important example of this (in the ship stabilization problem), we have those qualities of the effective waveslopes which are due to the course and speed of the ship. The apparent frequency of a given wavetrain will change as the ship's course and speed are changed, but in this change we can see a great deal of specific cause and effect. There may or may not be other qualities of the input for which a sufficiently strong cause-effect relationship can be found, e.g. the frequency of the waves vs. the strength of the wind.

Random Qualities: There are other properties of the input, however, in which the cause-effect relationship will be lost. These qualities can only be characterized in terms of

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their collective nature, i.e. in terms of their probabilities.
The true frequency distribution of waves in a given storm is an
example of such a quality.

Stationary and Quasi-Stationary Time Series: In studying
inputs, one often speaks of the stationary time series. By
that we mean a time series whose qualities are principally ran-
dom, and whose power spectrum does not change after a reasonable
length of time, i.e. for a sufficiently large sample. If a time
series is not stationary, but yet changes only slightly in
amounts of time comparable to the important time constants of
the system under discussion, the time series may be called
quasi-stationary and be treated as a stationary time series
which changes from time to time.

The effective waveslope input to a ship, cruising at will
over the ocean, is certainly not a stationary time series, but
it seems to be sufficiently well-behaved to be considered a
quasi-stationary time series.

C. Ways of Using Waveslope Information

In the preceding section we saw that the time series
which characterize effective waveslopes (i.e. which character-
ize the torques on the ship due to waves), may have both sys-
tematic and random properties. The presence of the systematic
properties, more than anything else, tends to make these time
series quasi-stationary rather than stationary. For example,
if the ship changes course, the properties of the time series

characterizing the effective waveslope change. It is clear that one could not adjust the parameters of the system before the ship went to sea, and have any hope that they would remain optimized (against input) for all the various situations in which the ship would later find itself. This leads us to the following comparison.

A Priori vs. A Posteriori System Adjustments: If one is dealing with a stationary time series it is possible to adjust the system to take account of this input at the outset, and then this adjustment remains optimum for all time. When, on the other hand, one is dealing with quasi-stationary time series, whose properties vary in a serious way, the system must be readjusted from time to time as the qualities of the time series change.

Now, in order to ascertain when and in what way the time series changes, a certain amount of observation must be carried on. Without observation one cannot become aware of the nature of the particular quasi-stationary time series one is dealing with. From this there follows a general theorem:

> The use of input information in the ship-stabilization problem appears to have meaning only in conjunction with: (1) a continuous or sampling observation of waveslopes, speed and course of ship, wind strength, etc.; and (2) a stabilization system certain of whose parameters may be adjusted from time to time in accordance with the findings of observation.

Adjusting for Systematic and Random Qualities: It is axiomatic that in optimizing systems with respect to their

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input, systematic properties must be adjusted for analytically, and random properties adjusted for statistically.[12] One first takes care of all the effects whose causes can be seen, and then treats the residue by statistical methods.

In the case of ship stabilization, we should, if at all possible, treat the changes of apparent frequency and effective waveslope due to course or speed, as due to such, and not as due to random causes. Perhaps the wind velocity and ocean region will also have significant systematic effects which can be accounted for and removed. The residue will not be an essentially stationary time series for all time, courses, speeds, etc., but it will certainly be infinitely more stationary than the original "raw" input.

In practice, one might go about "removing" (i.e. correcting for) the systematic properties of the effective waveslope input in something like the following way. Measure the speed of the ship, the strength of the wind, and the course of the ship relative to the oncoming waves, either by means of automatic devices or by human observation, or both. Then, assuming the system optimized at one course and speed, etc., analytic considerations will tell how the parameters of the system should be readjusted

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[12] This is emphasized by N. Wiener, Extrapolation, Interpolation, and Smoothing of Stationary Time Series, page 71. The "control chart" used in quality control is based on a similar idea.

as any of these variables change. Any residual deviation from optimum performance will now presumably be due to changes in the random component of the input. This is exactly the modus operandi of the "Following Sea" - "Beam Sea" switch in the Denny-Brown fin-control system, although it only accounts for the effect of changes in course. Such a system of adjustment can and should be made more or less sophisticated, according as experience shows more or less gain in performance when such and such a variable is corrected for.

D. The Statistical Approach to Servo Design

Let us assume for the sake of the argument, that all the known systematic effects have been accounted for leaving a residue which is essentially an almost-stationary, random time series. This time series may be expected to change somewhat with time, albeit slowly, so that over all time we must think in terms of a set of related time series, rather than in terms of single one. Such a set of related time series is known as an ensemble, and usually we strive to optimize the system in terms of the "average" properties of the ensemble.

The optimization of system performance in terms of such inputs may be accomplished by the methods of Wiener, et al.[13] These methods are based on a minimization of RMS "error",

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[13] Wiener, op. cit.

"error" being defined in general as some function of the difference between the actual behavior and the desired behavior.

Specification of Parameters vs. Specification of Function: Being variational in nature, these methods might be expected to specify the complete functional form of the optimum system. However, physically realizable systems are not free to be completely specified by an arbitrary input, and hence one must to some extent pre-specify the functional form of the system, and carry out the optimization in terms of the parameters of the pre-specification.[14] This is especially true in optimizing servomechanisms, for many constraints (of which the optimizing process is not a priori aware) act to narrow the range of allowable functional form and parameter values of a servo system. In the usual case, only a small proportion of the system parameters may be allowed to vary.

Definition of "Error": In the ship-stabilization problem "error" is obviously related to the difference between the desired angle of roll of the ship and the actual angle of roll. Suppose, for example, that we desire to totally stabilize against roll, then this difference is simply the residual roll, θ. It would seem reasonable to define "error" as equal to this residual roll. However, if we do this we do not take

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[14] See Wiener, op. cit., page 15, and James, et al, Theory of Servomechanisms, pp. 310-11.

account of the fact that even a moderate amount of roll <u>angle</u>
at the higher frequencies may cause intolerable roll <u>accelera-</u>
<u>tions</u>. Perhaps then, we should optimize in terms of $\dot{\theta}$, or
further, in terms of some weighted combination of θ and its
derivatives. But what weighting to choose? These are ques-
tions regarding the specification of "optimum", and are ques-
tions which must be answered by those naval architects, gunnery
experts, physiological-psychological experts, etc., who are
able to give qualified opinions on the most desirable <u>end</u>
<u>results</u> of ship stabilization.

<u>What and How to Adjust</u>: If the system is to be opti-
mized by adjusting certain pre-specified parameters, which
parameters shall be adjusted, and how was their pre-specifica-
tion arranged? The answer to this question is not so difficult.
In the first place the functional form of the regulated ele-
ment response is determined by the decision to use, for example,
a <u>single tank system</u>. This is what is known as a policy decis-
ion, as no purely analytic process could make this decision,
without <u>a priori</u> knowledge of the existence (and functional
form) of a single tank system. Some of the parameters of the
regulated elements are pretty well fixed by the original
parameters of the ship. Once the system is constructed none
of the parameters of the regulated elements, with the possible
exception of Q_t, can be adjusted to an appreciable degree
while at sea.

Hence it is clear that most of the adjustments must be made in the control system. Perhaps the most obvious adjustment is a change in the proportions of the acceleration, velocity, and acceleration control signals. Quite a number of other adjustments -- more or less filtering of the signal, more or less use of "lead" and "lag", etc. -- can be envisioned, depending on the complexity of the control.

The utility of the frequency-response approach is emphasized by the above remarks, for obviously, it is our guide and aid in making decisions as to functional form, such as the decision to use diversified vs. single tanks, etc.

VI. SUMMARY OF RESULTS AND CONCLUSIONS

This part of the report has been concerned with the lin-earized theory of performance, i.e. with the performance which may be expected from a system whose capacity is never exceeded. It has had the double object: (a) of finding high-performance systems; and (b) of showing the utility of servo methods in ship-stabilization work. The satisfactory accomplishment of (a) is felt to be a sufficient demonstration of (b).

Paralleling Part A, we have studied the high-performance problem in terms of three factors: (1) regulated elements; (2) regulating elements; and (3) the input. These three parts of the problem have been taken up in Chapters III, IV, and V, re-spectively: the regulated and regulating elements being analyzed and synthesized by frequency-response methods; and the use of in-put information being related to statistical, error-minimization methods.

A. Frequency-Response Approach to Ship-Tank Design

In preliminary work it was found that the linearized per-formance of the single tank system depended on only four criti-cal non-dimensional parameters: Q_t, λ_t, Ω_t, and Ω_{st}. The purpose of Chapter III was to find the best values for these four parameters, and further to study the possibilities of various diversified tank systems. An acceleration only control was used throughout this chapter.

Single Tank System: The results for the single tank system may be summarized as follows: The tank damping parameter, Q_t, should be as low as possible, about 0.2 - 0.1. The tank strength (and weight) parameter, λ_t, can be as low as practical limitations permit, without detriment to the linearized performance, probably not less than 0.10. The tank relative frequency, Ω_t, should be close to 1.0, if anything a little above. The secondary resonance relative frequency, Ω_{st}, should be as high as it is practical to make it, approaching something like 3.0.

A finite secondary resonance frequency was found to be on the whole disadvantageous for the single tank system. However, a system with such secondary resonance actually stabilizes the frequencies above the frequency at which secondary resonance occurs more effectively than a single tank system with $\Omega_{st} =$ infinity. This fact pointed the way to an interesting diversified tank system.

Diversified Tank Systems: By diversified tank systems we mean a ship in which several U-tube tanks (with different parameters) are used to produce the stabilizing action. Two double-tank systems were considered: a "stagger-tuned" arrangement; and a single tank (with finite Ω_{st}) aided by a small "helping" tank at the secondary resonance null. The first arrangement provides little improvement, but the second one is of more interest.

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By means of this second arrangement, the high-frequency end of the loop-transfer locus can be made to lie wholly in the right hand plane. This is indicative of a system with especially effective stabilization at the high frequencies. This result is essentially due to the fact that this system utilizes the inertia of the water, rather than the weight of the water, to produce high-frequency stabilizing moments. While this second diversified tank system produces improvement of the high-frequency response, the gain continues to be limited by the unimproved end of the locus. Its full effectiveness can only be realized by using additional control signals to improve the low-frequency response. The use of these various controls was discussed in the following chapter.

B. Frequency-Response Approach to Control Design

We investigated four more or less distinct control problems: (a) the use of position and velocity to augment the acceleration control; (b) the problem of control lag; (c) the use of "lead" circuits; and (d) the use of open-cycle control or "feedahead".

Position and Velocity as Complementary Signals: When the acceleration signal is assisted by a position signal the very-low-frequency response is improved but there occurs a position-acceleration null exactly analogous to the secondary resonance null! This null may be filled in by a velocity signal, which

improves the low-frequency end of the loop-transfer locus in the same way that the "helping" tank improves the high-frequency end of the locus.

When acceleration, velocity, and position signal control is used on a single tank system (with Ω_{st} = infinity), the gain is limited by the unimproved, high-frequency end of the locus. This is analogous to the double tank case described above. Now when this augmented control is used on a double tank system, both ends of the loop-transfer locus are improved, and for ideal controls, the whole locus lies in the right hand plane. A system with this locus would effectively stabilize all frequencies. Actually, one cannot completely realize this locus as the response of the control system must eventually lag and drop off. This will cut off the stabilizing action at some rather high frequency, but notice that the cutoff lies wholly in relatively light and flexible controls and not in the regulated elements!

Thus Chapter IV completes the step-by-step improvement process begun in Chapter III. The various modifications of the loop-transfer locus have been shown previously in Nyquist diagrams. The following figure, Figure 31, shows these modifications in terms of the amplitude of the loop-transfer locus versus relative frequency. We see that the performance of the final system is limited only by control lags (and its capacity, of course). This emphasizes the importance of control lags.
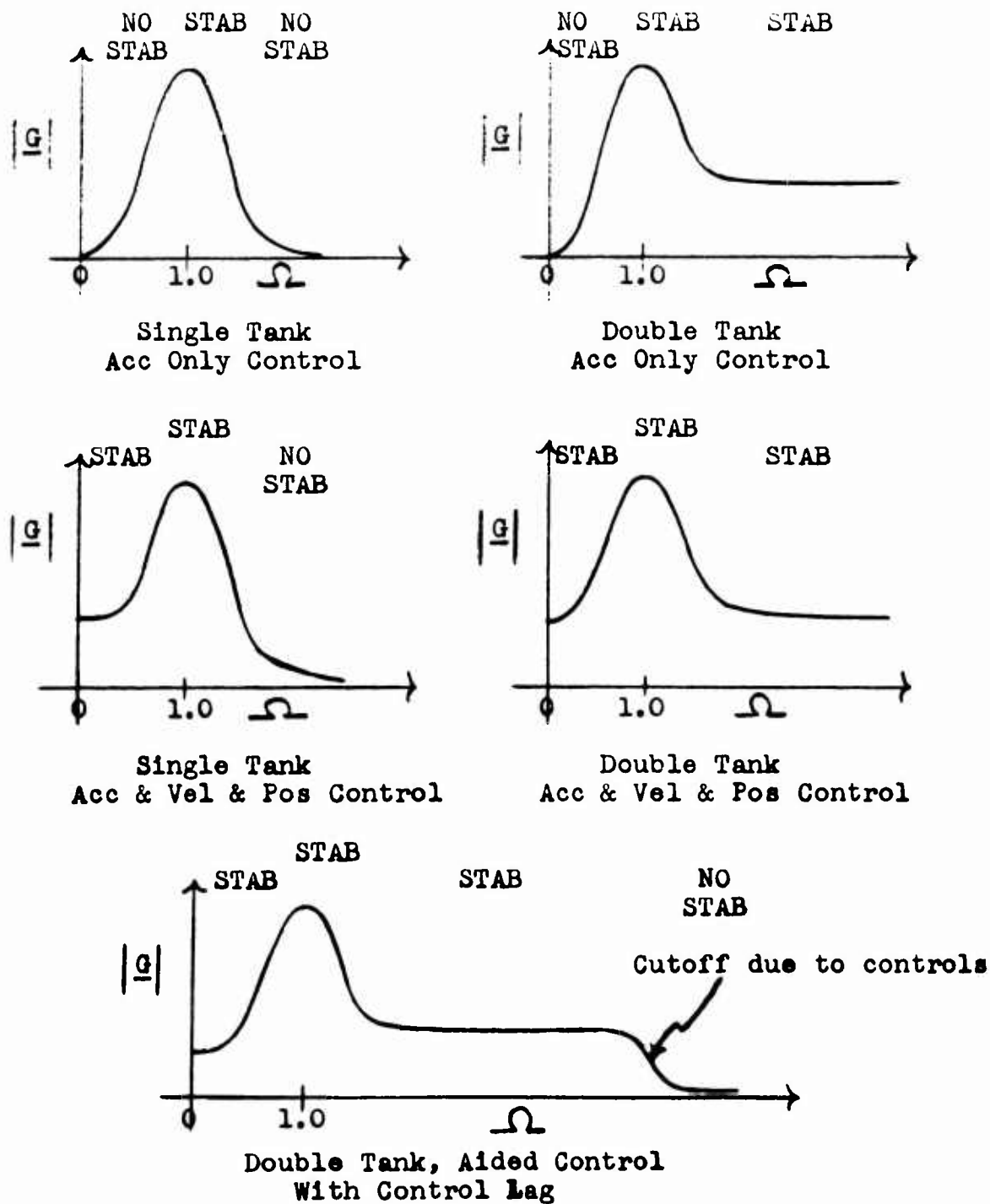
Figure 31. Step-by-Step Modification of the Loop-Transfer
Locus and Corresponding Stabilization

Control Lags: Their Origin and Their Effect: To facilitate the analysis of real controls, they were broken down into three sub-blocks: (1) the sensitive elements; (2) the control amplifier; and (3) the positioning motor. From the various arguments put forward, it appeared that the principal lag in the control response should be due to the relatively heavy positioning motor. Figure 28 illustrated the effect of rather severe lag in the positioning motor block.

Uses of Lead Circuits: Figure 28 also demonstrated that a "lead" circuit could be used to improve the response of lagging controls. Such a circuit, being realizable, is more realistic than a "differentiator". It provides signal plus derivative control over a moderate range of frequencies, or from the locus modification point of view it inserts a phase lead "bump" in the locus where such a bump is needed.

"Feedahead" or Open Loop Control: Most of the previous discussions were of controls as elements in a feedback loop. It is known in the servo art, that in certain cases the feedback performance of a system may be considerably augmented and improved by the simultaneous use of "feedahead" or open loop control. This was illustrated in Figure 30. The interesting part of the result is that the two actions multiply. Feedahead also may provide complete stabilization at one or more frequencies. Remembering that our overall stabilization (without feedahead) was equal to the product of passive

stabilization and feedback action, we may set up the following
historical sequence, which holds for stabilization by any means.

$$\frac{\theta}{\psi} = \underline{C} \qquad\qquad\qquad = \text{(passive stabilization)}$$

$$\frac{\theta}{\psi} = \underline{C} \times (\frac{1}{1 + \underline{A}\,\underline{B}}) \qquad = \text{(passive stab) x (feedback stab)}$$

$$\frac{\theta}{\psi} = \underline{C} \times (\frac{1}{1 + \underline{A}\,\underline{B}}) \times (1 - \frac{X}{\underline{C}\,\underline{B}^{-1}}) = \begin{matrix}\text{(passive stab) x (feedback stab)} \\ \text{x (feedahead stab)}\end{matrix}$$

The first and second of these schemes each have been used
with stabilization by tanks, gyros, and fins; the third scheme
is yet to be utilized for ship stabilization. For the proper
operation of a feedahead control it is necessary to measure the
effective waveslope, distinct from the motion of the ship. This
is a difficult measurement which would require balancing a sig-
nal from pressure detectors on the hull, against various signals
corresponding to the motion of the ship. However, the measure-
ment does not seem out of the question at the moment, and once
made, would provide an interesting signal, useful for oceano-
graphic as well as for stabilization purposes.

C. The Use of Waveslope Information in Design

Most of the work in this Part C of the report is based on
the so-called frequency-response approach to servo design.
While this approach is extremely useful it does not provide
an explicit means by which the nature of input may be used to
improve the effectiveness of a stabilization system. This can

only be done by other methods, the choice of these methods depending on the nature of the input.

Mathematical Nature of the Input: The effective waveslope input is basically a time series. As such it appears to have two different types of qualities: (1) systematic; and (2) random. For example, changes in the course of the ship (relative to the waves) produce large and systematic changes in the effective waveslope time series. Hence this time series is far from stationary.

Ways of Using Waveslope Information: If one removes the effect of systematic variations in the input time series, one is left with a much more stationary, random residue, which may be used for statistical optimization methods. However, the systematic effects can only be known and removed if the quantities on which they depend (ship's course, ship's speed, wind strength, etc.) are also known at any time, or sampled from time to time. For this reason the use of input information for optimization has meaning only in conjunction with a means for observing these quantities and for adjusting the system parameters accordingly. As an example, cite Denny-Brown's human observer and "Following Sea"-"Beam Sea" switch.

The Statistical Approach to Servo Design: Once the systematic effects have been removed, the residue may be treated by the RMS "error" minimization methods of Wiener, and others. In practice this optimization must be done in

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terms of _parameters_ rather than _functions_, hence the utility of the frequency-response approach as a first approach. The definition of "error" determines the manner in which the parameters will be adjusted, and brings up the question of just what is the most desirable residual motion. This question is almost unanswered at the moment and would seem to deserve some consideration (by competent experts) in the not-too-distant future. While theoretically, any of the system parameters could be adjusted, practical limitations will probably necessitate that most of the adjustment be in the parameters of the control system.

D. Conclusions

In this Part C, we have proceeded step-by-step to find ways and means by which systems of greater and greater effectiveness might be achieved. While the results of this part are based on the assumption that the capacity of the system is never exceeded, practical questions have not been neglected. Parameters have never been assumed to have values which could not be attained in the present state of the art, and the important question of lag has been discussed in some detail. The end results of these studies are quite interesting.

How High is "High-Performance": At the end of Section A of Chapter IV we achieved (in the double tank with aided control) a stabilization which amounted to something like a 95%

reduction of "average" rolling, and a correspondingly greater

reduction of pure resonance rolling. Quantitative numerical

predictions of high performance were not carried out beyond

this point, but the feedahead and statistical optimization

discussed in later chapters would both tend to further improve

the above result.

Grounds for Belief: In light of the fact that this kind of

stabilization is a number of times better than anything which

has yet been achieved in practice by any means, it may be well

to examine what, if any, grounds one has for believing that

such performance can actually be realized. There are a number

of plausible objections to this belief.

It has been said that the stabilizing action of tanks is

inherently slow, hence their high-frequency response must be

poor. This is true for a stabilizing action based only on the

weight and hence the displacement of the water. This is not

true if the stabilizing action is also made to depend on the

inertia and hence acceleration of the water. It is a clean-

cut, experimentally verified result of the Stanford research,

that the stabilizing action can easily be made to depend on

inertia, and that the resulting torques have the desired sense.

From this it follows that the response of a properly designed

stabilization system using tanks, can be as rapid or more rapid

than that of any other known type of ship-stabilization system

(e.g. fins).

It might be said that the results of the trials of the USS Peregrine constitute a negative result for stabilization by tanks.[15] However, the known shortcomings of the Peregrine design appear to constitute a sufficient explanation of its performance. The author is not aware of any way in which the Peregrine trials contradict the basic results of the Stanford research; and there are, of course, a number of ways in which these trials underline and emphasize the results presented in this report.

It might be pointed out that nothing even approaching this kind of performance (95% stabilization) has yet been achieved in practice. This is true, but not necessarily indicative. The Denny-Brown fin stabilization system, for example, appears to produce about 60% stabilization when operating within its capacity.[16] Now while there is a factor of _eight_ between a 5% residual and a 40% residual, there also was a factor of about _four_ between the stabilized performance of the _double tank with aided control_ and the less spectacular performance of the _single tank with aided control_. This last single tank system is believed to be a more thoroughly optimized design than the Denny-Brown fin system (as of 1945), hence the performance of the Denny-Brown system appears to be about what one would expect from the results of this report.

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15 These trials have been discussed by the author in an unpublished memorandum, "Trials of the USS Peregrine- Report and Recommendations", December 19, 1949.

16 See J. F. Allan, "The Stabilization of Ships by Activated Fins," Trans. Inst. Naval Arch. (1945).

In addition to the above discussion of objections, there are several other more direct grounds which strengthen our belief in the results of this part. In the first place, these results are based on the theory developed in Part A and experimentally verified by model tests in Part B. Unlike a real ship, the model was constrained in such a way as to have only one degree of freedom, but various theoretical and practical considerations lead us to believe, that this will have only a second-order effect on predictions made from the model results. The work in Part C represents little extrapolation of the basic theory, rather it is a rational and organized use of these results. We have tried to avoid assuming anything more about any part of the system than is justified by the known state of the art, whether such part be in the regulated elements or in the regulating elements.

It is possible that the predictions of this Part C will later be found to have been slightly optimistic or even slightly pessimistic, but we believe that they will not be found grossly wide of the mark.

Control Versus Capacity: Part C emphasizes the role of controls in achieving high-performance stabilization. Clearly the difference in capacity required in a system which stabilizes 80% of the roll and one which stabilizes 95% of the roll is not great, but there is a ratio of four in residual motions! Thus high performance against a given waveslope is chiefly a

function of the design of the controls, and the design, but not the strength of the regulated elements.

Notice that improved performance due to improved controls or improved regulated element design, comes almost free of charge. The principal cost of a stabilizing system is in the weight and expense of the stabilizing device. Sophisticated controls only "cost" if they decrease the reliability of the system, and this need not be the case.

From the results of this Part C, it appears that the control problem (which is essentially what we have studied here) will not be the limiting factor in the ship stabilization. Rather, the most severe limitation is likely to come from economic considerations which will put a ceiling on the capacity of the system. No system can provide high performance when its capacity is exceeded by an appreciable percentage.

Rational Design Versus Empirical Design: Perhaps the most important result of Part C, is the way in which it illustrates the utility of using a highly rational approach to the problem. The author believes that it would be extremely difficult if not impossible to arrive at the results of this part by any experimental or empirical method. The adjustment of the four parameters of the single tank system alone, would represent an almost herculean task, and probably cost a small fortune, if it had to be done by physically adjusting the system, even if a model system were used; and it seems entirely out of the question to optimize a full scale tank system empirically.

Part C, by achieving its goal to a considerable degree, gives the capacity limitations of the system greater importance thereby. These limitations and other practical questions are discussed in the following Part D.

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greater effectiveness (for a given capacity). It appears that tank systems using inertia effects, may be as rapid in response as any other known stabilization systems. Results indicate that as long as the system's capacity is not exceeded, it should be possible to achieve greater than 90% stabilization. The three factors in ship stabilization: regulated elements; regulating elements; and the input, are discussed in turn, using appropriate servo methods and concepts. This provides considerable grounds for believing that high-performance stabilization systems using tanks (and presumably other devices) can be achieved. It shows rational techniques by which such stabilization systems may be designed, and indirectly it emphasizes inherent difficulties in design procedures which do not utilize servo techniques.